Graded properties of t-norms

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T-norms and other binary aggregation operators can be viewed as functions $[0,1]^2 \to [0,1]$, or equivalently as binary fuzzy relations on $[0,1]$. Consequently, graded properties of fuzzy relations as studied in [1, 2] are applicable to t-norms; moreover, the defining properties of t-norms themselves can consistently be made graded by replacing crisp relations $=, \leq$ by residual operations $\Rightarrow, \Leftrightarrow$ and re-interpreting classical logical symbols by fuzzy ones: e.g., the degree of commutativity of T is defined as $\bigwedge_{\alpha\beta}(T\alpha\beta \Leftrightarrow T\beta\alpha)$. These graded properties can conveniently be handled by first-order fuzzy logic [3], esp. within the framework of Fuzzy Class Theory [4].

A graded theory of t-norms has been introduced in [5]; here we present some recent advances in the topic. In particular, we present results on the graded properties of commutativity, associativity, idempotence, monotony, generalized Lipschitz property, unit and null elements, and on preservation of these properties under graded equality and functional composition.

References

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