

GENERALIZED CONGRUENCE AND CLOSURES: BASIC PROPERTIES AND APPLICATIONS TO FUZZY RULE-BASED SYSTEMS

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Images of fuzzy sets under fuzzy relations have been investigated mainly in two contexts: On the one hand, mostly under the terms “full image” or “direct image” [3], they can be regarded as very general tools for fuzzy inference [1, 3]. On the other hand, under the term “extensional hull”, the image of a fuzzy set under a fuzzy equivalence relation yields the smallest superset which is “closed” under the relation, where this property is usually called “extensionality” [4].

In the first part of this contribution, a general concept of closedness under a fuzzy relation is introduced along with an appropriate closure operator, naturally extending the well-known concepts of extensionality and extensional hulls, respectively.

The second part is devoted to adding another view on hull/image operators. We will see that these operators can be used to model linguistic modifiers which may be useful for reducing the size of a fuzzy rule bases while maintaining or even improving interpretability and expressiveness.

References

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