Monotonicity in Vague Environments

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Zadeh's famous extension principle [8, 9, 10], as a general methodology for extending crisp concepts to fuzzy sets, has served as the basis for the inception of new disciplines like fuzzy analysis, fuzzy algebra, fuzzy topology, and several others. Most importantly, this fundamental principle allows to extend crisp mappings to fuzzy sets. Another well-known application—which is particularly important in fuzzy decision analysis and fuzzy control [4, 6]—is the possibility to define ordering relations for fuzzy sets.

This contribution is devoted to links between these two fields: we study in which way the monotonicity of a mapping is preserved by its extension to fuzzy sets. However, we do not restrict to the well-known methodology of extending crisp orderings to fuzzy sets, but we start from the more general case that the domain under consideration is equipped with a fuzzy ordering [1, 2] induced by some non-trivial fuzzy concept of indistinguishability. Even in such a case, it is possible to define orderings of fuzzy sets in a way similar to the extension principle [1, 3].

First, we consider the classical case of orderings of fuzzy sets defined from crisp orderings by means of the extension principle. We show that the monotonicity of a mapping directly transfers to its extension. Furthermore, the same holds for the componentwise monotonicity of n-ary operations. Next, we consider the general case that the universe is equipped with a fuzzy ordering induced by a fuzzy equivalence relation E. It is proved that the monotonicity of a mapping φ is preserved by its extension if and only if φ is extensional with respect to E [5, 7]. However, it turns out that an analogous correspondence does not necessarily hold for n-ary operations.

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