

# Applications of Fuzzy Orderings: An Overview

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There are two most fundamental relational concepts in mathematics which accompany mathematicians as well as computer scientists and engineers throughout their life in science—*equivalence relations* (reflexive, symmetric, and transitive relations) and *(partial) orderings* (reflexive, antisymmetric, and transitive relations).

It is not surprising that, within the early gold rush of fuzzification of virtually any classical mathematical concept, these two fundamental types of relations did not have to await the introduction of their fuzzy counterparts for a long time [22].

Fuzzy equivalence relations are now well-accepted concepts for expressing equivalence/equality in vague environments [8, 13, 16, 18, 20, 21] (in contrast to Zadeh’s original definition, now with the additional degree of freedom that the conjunction in transitivity may be modeled by an arbitrary triangular norm [15]).

In the meantime, fuzzy equivalence relations have turned out to be helpful tools in various disciplines, in particular, as soon as the interpretation of fuzzy sets, partitions, and controllers [16, 21, 10, 14] is concerned. More direct practical applications have emerged in flexible query systems [12, 17] and fuzzy databases in general [19].

Fuzzy (partial) orderings have been introduced more or less in parallel with fuzzy equivalence relations [22], however, they have never played a significant role in real-world applications.

This paper advocates a “similarity-based” generalization of fuzzy orderings, however, not from the pure mathematical viewpoint of logic or algebra (for what we would like to refer to the extensive studies in [2, 3, 5, 11]). Instead,

we attempt to demonstrate the potential for applications by means of considering comprehensive overviews of four case studies. Those are flexible query systems [7], ordering-based modifiers [1, 9], and orderings of fuzzy sets [4]. Finally, we also discuss the interpretability property, for which orderings of fuzzy sets are of fundamental importance [6].

## Acknowledgements

The author gratefully acknowledges support of the *Kplus Competence Center Program* which is funded by the Austrian Government, the Province of Upper Austria, and the Chamber of Commerce of Upper Austria.

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