Similarity-Based Fuzzy Orderings: Recent Advances

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Fuzzy orderings with reference to an underlying context of similarity were first defined by Höhle and Blanchard [9]. Since then, several authors have contributed to the development of this class of fuzzy relations, both from the theoretical and the practical side (e.g. [1–4, 7, 8]). Two problems remained open throughout several years: how to define strict fuzzy orderings and how to define lexicographic compositions of fuzzy orderings. It is clear, of course, that these two open problems are strongly related, since lexicographic composition is only possible if a meaningful way to extract the strict part of a fuzzy ordering is available.

This contribution aims at presenting an overview of recent advances in this direction. Firstly, we summarize the content of a recent paper devoted to strict fuzzy orderings in the similarity-based framework [5]. We introduce and justify a similarity-based concept of strict fuzzy orderings and provide constructions how fuzzy orderings can be transformed into strict fuzzy orderings and vice versa. We demonstrate that there is a meaningful correspondence between fuzzy orderings and strict fuzzy orderings. Unlike the classical case, however, we do not obtain a general one-to-one correspondence. We observe that the strongest results are achieved if the underlying t-norm induces a strong negation, which, in particular, includes nilpotent t-norms and the nilpotent minimum.

Secondly, we highlight the results of recent research on lexicographic composition of similarity-based fuzzy orderings. Starting from the approach to strict fuzzy orderings presented in [5], we provide a construction principle for lexicographic composition of fuzzy orderings. Note, however, that an approach to lexicographic composition is only meaningful if the lexicographic composition of linear orderings is linear again. Therefore, we approach the question if (or under which conditions) this fundamental property holds, assuming that T-linearity [6, 9] is the appropriate concept to express linearity of fuzzy orderings. It turns out that, once again, we only obtain meaningful results if the underlying t-norm induces a strong negation.

References

[1] R. Bělohlávek. *Fuzzy Relational Systems. Foundations and Principles.* IFSR Int. Series on Systems Science and Engineering. Kluwer Academic/Plenum Publishers, New York, 2002.

[2] U. Bodenhofer. A similarity-based generalization of fuzzy orderings preserving the classical axioms. *Internat. J. Uncertain. Fuzziness Knowledge-Based Systems*, 8(5):593–610, 2000.

[3] U. Bodenhofer. Representations and constructions of similarity-based fuzzy orderings. *Fuzzy Sets and Systems*, 137(1):113–136, 2003.

[4] U. Bodenhofer. Applications of fuzzy orderings: An overview. In K. T. Atanassov, O. Hryniewicz, and J. Kacprzyk, editors, *Soft Computing. Foundations and Theoretical Aspects*, pages 81–95. EXIT, Warsaw, 2004.

[5] U. Bodenhofer and M. Demirci. Strict fuzzy orderings in a similarity-based setting. In *Proc. Joint 4th Conf. of the European Society for Fuzzy Logic and Technology and 11 Recontres Francophones sur la Logique Floue et ses Applications*, pages 297–302, Barcelona, September 2005.

[6] U. Bodenhofer and F. Klawonn. A formal study of linearity axioms for fuzzy orderings. *Fuzzy Sets and Systems*, 145(3):323–354, 2004.

[7] M. Demirci. A theory of vague lattices based on many-valued equivalence relations—I: general representation results. *Fuzzy Sets and Systems*, 151(3):437–472, 2005.

[8] M. Demirci. A theory of vague lattices based on many-valued equivalence relations—II: complete lattices. *Fuzzy Sets and Systems*, 151(3):473–489, 2005.

[9] U. Höhle and N. Blanchard. Partial ordering in L-underdeterminate sets. *Inform. Sci.*, 35:133–144, 1985.