Opening and Closure Operators of Fuzzy Relations: Basic Properties and Applications to Fuzzy Rule-Based Systems

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Images of fuzzy sets under fuzzy relations have been investigated mainly in two contexts: On the one hand, mostly under the term "full image" [5], they can be regarded as very general tools for fuzzy inference, leading to the so-called "compositional rule of inference" [1, 5]. On the other hand, under the term "extensional hull", the image of a fuzzy set under a fuzzy equivalence relation yields the smallest superset which is "closed" under the relation, where this property is usually called "extensionality" [6].

In the first part of this contribution, after recalling some basic definitions and properties, we propose a new generalized concept of closedness under a fuzzy relation (let us call it "congruence") which naturally extends the notion of extensionality to arbitrary binary fuzzy relations. Based on these considerations, under the assumption of T-transitivity, we are able to give explicit formulae for the congruent opening, i.e. the largest congruent subset, and the congruent closing, i.e. the smallest congruent superset, of a fuzzy set. It will turn out that this directly leads to full images—as already known for fuzzy equivalence relations.

The second part is devoted to a new view on images of fuzzy sets under fuzzy relations—by integration of the results on congruence and the inference-based interpretation of images under fuzzy relations, we are able to provide a new framework for defining linguistic modifiers, both ordering-based ones like "at least", "at most", or "between" and usual weakening and intensifying ones like "more or less", "roughly", or "very".

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References

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