Transitivity-Preserving Aggregation of Fuzzy Relations

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Aggregation is a fundamental process in decision making and in any other discipline where the fusion of different pieces of information is of vital interest. In fuzzy decision analysis, the aggregation of truth values from the unit interval is particularly important and has been studied extensively (see [3] for a state-of-the-art overview)

While almost all studies have been focused on algebraic properties of aggregation operators (e.g. associativity), this contribution is devoted to a different aspect—the preservation of T-transitivity. The basic motivation for this particular study comes from fuzzy querying.

Flexible (fuzzy) querying systems are usually designed not just to give results that match a query exactly, but to give a list of possible answers ranked by their closeness to the query—which is particularly beneficial if no record in the database matches the query in an exact way [7]. The closeness of a single value of a record to the respective value in the query is usually measured using a fuzzy equivalence relation, that is, a reflexive, symmetric and T-transitive fuzzy relation. Recently, a generalization has been proposed [2] which also allows flexible interpretation of ordinal queries (such as "at least" and "at most") by using fuzzy orderings [1]. In any case, if a query consists of at least two expressions that are to be interpreted vaguely, it is necessary to combine the degrees of matching with respect to the different fields in order to obtain an overall degree of matching. Assume that we have a query $(q_1, \ldots, q_n)$, where each $q_i \in X_i$ is a value referring to the i-th field of the query. Given a data record $(x_1, \ldots, x_n)$ such that $x_i \in X_i$ for all $i = 1, \ldots, n$, the overall degree

of matching is computed as

$$\tilde{R}((q_1, \ldots, q_n), (x_1, \ldots, x_n)) = \mathbf{A}(R_1(q_1, x_1), \ldots, R_n(q_n, x_n)),$$

where each R_i is a T-transitive binary fuzzy relation on X_i which measures the degree to which the value x_i matches the query value q_i. If R_i is a fuzzy equivalence relation, R_i measures the degree of similarity between q_i and x_i. If R_i is a fuzzy ordering, R_i gives the value to which x_i is smaller/greater than or similar to q_i.

It appears natural to require that the function $\mathbf{A}$ is an aggregation operator [3, 4, 6]. Moreover, it would be desirable that $\tilde{R}$ is still T-transitive in order to have a clear interpretation of the aggregated fuzzy relation $\tilde{R}$. Therefore, it is necessary to study which aggregation operators are able to guarantee that $\tilde{R}$ maintains T-transitivity. In this contribution, we put together pieces of existing knowledge and introduce some new results concerning the preservation of transitivity:

1. (weighted) dominating t-norms [5, 6]
2. (weighted) quasi-arithmetic means [4]
3. min-transitivity-preserving weighting schemes

References

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