Towards Modeling Dynamic Behavior with Integrated Qualitative Spatial Relations^{*}

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Abstract. Situation awareness and geographic information systems in dynamic spatial systems such as road traffic management (RTM) aim to detect and predict critical situations on the basis of relations between entities. Such relations are described by qualitative calculi, each of them focusing on a certain aspect (e.g., topology). Since these calculi are defined isolated from each other, dependencies between then are not explicitly modeled. We argue, that a taxonomy—containing a plethora of special cases of inter-calculi dependencies—can only be defined in a consistent manner, if evolution of entities and the relations of calculi are grounded in a unified model. In this paper, we define such a unified model, which is used to derive a taxonomy of inter-calculi dependency constraints contained in an ontology utilizing various spatial calculi. The applicability of this approach is demonstrated with a case study in RTM, and concluded with lessons learned from a prototypical implementation.

1 Introduction

Situation awareness in dynamic spatial systems. Situation awareness and geographic information systems (GIS) are gaining increasing importance in dynamic spatial systems such as road traffic management (RTM). The main goal is to support human operators in assessing current situations and, particularly, in predicting possible future ones in order to take appropriate actions pro-actively. The underlying data describing real-world entities (e.g., tunnel) and their spatial relations (e.g., inside, near), which together define relevant situations (e.g., a traffic jam inside and near the boundary of a tunnel), are often highly dynamic and vague. As a consequence reliable numerical values are hard to obtain, which makes *qualitative modeling* approaches better suited than quantitative ones [17]. Dynamic behavior in qualitative spatial calculi. Recently, ontology-driven situation awareness techniques [1], [6] and qualitative approaches to modeling the dynamic behavior of spatial systems [3] have emerged as a basis for predicting critical situations from relations between objects. Such relations are expressed by employing multiple *relation calculi*, each of them focusing on a certain aspect, such as topology [8], [20], size [13], or distance [15]. These calculi are often

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temporalized by means of *Conceptual Neighborhood Graphs* (CNGs, [9]) and dominance spaces [10], [11], imposing constraints on the existence of direct transitions between relations. The domain-independent nature of calculi and their focus on a particular aspect of relationship (e.g., topology), however, results in dependencies between calculi being not explicitly modeled (e.g., topological transitions imply transitions in the distance between object boundaries). In a pioneering work on qualitative distance, Clementini stresses the importance of interdependency between calculi as follows: "the meaning of close depends not only on the actual relative position of both objects, but also [on] their relative sizes and other scale-dependent factors" [5].

A unified model for inter-calculi dependencies. Ontology-based approaches to dynamic spatial systems utilizing multiple calculi tackle the integration of these calculi by providing dedicated modeling primitives, for instance in terms of so-called *axioms of interaction* [3] and *relation interdependencies* [2]. Since these approaches, however, completely abstract from the underlying continuous space, a taxonomy exhaustively describing inter-calculi dependencies is still missing. We argue, that such a taxonomy—containing a plethora of special cases of inter-calculi dependencies—can only be defined in a consistent manner, if evolution of spatial primitives (e. g., regions) and the relations of calculi are grounded in a unified model. In order to define such a unified model for motion and scaling of spatial primitives and their effects in terms of transitions in topology, size, and distance, we base on Galton's approach to constructing a "homomorphic image of the full space of possible region-pairs" [11].

Structure of the paper. In the next section, the focus of this work is detailed as the basis for discussing relevant related work. In Sect. 3, a unified model for spatial calculi is presented along with a case study in the domain of RTM. The model is the basis for an ontology briefly sketched in Sect. 4. Finally, Sect. 5 concludes the paper with lessons learned from a prototypical implementation.

2 Related Work

In this section, we discuss related work on modeling the dynamic behavior of spatial systems with qualitative spatial reasoning approaches, focusing on those approaches in the domain of GIS. In this discussion, we follow the common ontological distinction (cf. the SNAP/SPAN approach [14]) often applied in GIS [12], [23] between the states of a system describing relations between entities from a snapshot point-of-view, and the evolution between these states in terms of occurrents, such as events and actions. Causal relations between states and occurrents [12] comprise (i) qualification constraints defining preconditions for states (i. e., states enable or disable other states, e. g., being smaller enables being a part), and for occurrents (i. e., states allow or prevent occurrents, e. g., having very close boundaries enables becoming externally connected), whereas (ii) frame constraints define effects of occurrents¹ (i. e., occurrents cause other

¹ Ooccurrents initiating and terminating states (e.g., becoming very close initiates being very close) are not considered here, since they are modeled in CNGs.

occurrents, e.g., motion causes two objects becoming disrelated). In this paper, we focus on qualification constraints for states and occurrents, since these are the primary source of inter-calculi dependencies.

Many qualitative spatial reasoning approaches (e.g., [7], [10], [11], [21]) provide or utilize a single qualitative spatial calculus modeling a particular aspect, and naturally, encode qualification constraints in CNGs (i. e., each relation is a qualification constraint for its neighboring relations). A slightly broader view is applied in GIS [8], informally discussing states, in particular the size of objects, as qualification constraints for relations. The same constraint is used in a modeling framework for dynamic spatial systems [4] as qualification constraint on the transitions between relations. Arbitrary qualification constraints spanning multiple qualitative spatial calculi are explicitly supported in Bhatt's approach to modeling the dynamic behavior of spatial systems [3] in the form of so-called *axioms of interaction*. However, this modeling approach lacks a taxonomy of states and constraints. As a consequence, both must be provided by users of this modeling framework, instead of being integrated within its ontology.

Focusing on the integration of multiple calculi, Gerevini and Renz [13] discuss interdependencies between the Region Connection Calculus (RCC) and their Point Algebra for describing size relations. These interdependencies describe qualification constraints for states (i. e., relations) of one calculus in terms of states of the other. For example, a relation TPP (tangential proper part) of RCC entails a size relation < (i. e., the contained entity must be smaller than the containing one). Using the same calculi (RCC and size), Klippel et al. [18] investigated the impact of different size relationships on the relation transitions in RCC induced by motion events, and the cognitive adequacy of these changes. Since the interdependencies between topological and size relations are rather obvious, providing a formal integration model, however, has not been the focus.

Clementini et al. [5] present several algorithms for combining distance and orientation relations from a compositional point-of-view (e.g., these algorithms compute the composition of distance relations, given a known orientation relation). In contrast, we focus on interpreting relations of a particular calculus as qualification constraints for relations and/or transitions in other calculi.

In summary, existing works lack a model of space and of spatial primitive pairs, preventing consistent integration of multiple calculi with major evolution causes (motion, scaling, orientation, shape, cf. [8]). In the next section, we discuss such a model along three spatial calculi modeling important aspects like topology (RCC, [20]), distance of boundaries [15], and size [13].

3 Inter-Calculi Dependencies in Spatial Calculi

In qualitative spatial reasoning, as introduced above, a multitude of different spatial calculi has been proposed. Although each of these calculi focuses on a particular aspect of the real world, some of their relations implicitly model other aspects as well (i. e., these relations restrict the relations that can hold and the transitions that can occur in another calculus). For instance, a topological nontangential proper part relation (NTPP) between two objects does not only define that a particular object is contained in another one, but also implicitly defines that the contained object must be smaller than the containing one [13]. Additionally, real-world evolution abstracted to transitions in one calculus might be modeled in more detail in another calculus. For example, a topological transition from being disconnected (DC) to being externally connected (EC) in RCC is modeled from a distance viewpoint [15] with a sequence of relations and transitions, comprising transitions from being very far (VF) over far (F) and close (C) to being very close (VC). We make such assumptions explicit by combining exisiting calculi with qualification constraints modeling inter-calculi dependencies.

In order to define such qualification constraints in a consistent manner and account for a plethora of different special cases, a mapping between relations and the underlying spatial primitives including their numerical representation is needed. For example, let us consider relations describing the spatial distance between object boundaries. Since the boundary of an object implicitly defines its size and center, the options concerning the distance between the boundaries of two objects can only be narrowed by taking into account information about their topological relationship, relative size, and distance of their centers: If one object is known to be a proper part of the other one, a rather small object being located at the center of a large object is regarded to be very far from the large object's boundaries, whereas the same object with a large distance to the center would result in the boundaries being considered to be very close. The boundaries of two nearly equally-sized objects would be considered very close as well.

As the basis for determining the above sketched variety of special cases making up inter-calculi dependencies, we base upon Galton's approach [11] to deriving a two-dimensional image of relations from the CNG of RCC, since this approach covers the full space of possible region-pairs. In such a two-dimensional image, the topological relations between two spheres are encoded, using the radii r1 and r2 of the spheres along the x-axis (x = r1/(r1+r2)) and the distance d between their centers on the y-axis (d/2(r1+r2)). The relations DC (disconnected), EC (externally connected), PO (partly overlapping), TPP (tangential proper part) and its inverse TPPi, NTPP (non-tangential proper part) and its inverse NTPPi, as well as EQ (equals) are defined in terms of these two measures in (1).

DC
$$0.5 < y < 1$$
 EC $y = 0.5$
PO $|0.5 - x| < y < 0.5$ EQ $x = 0.5 \land y = 0$
TPP $0 < y = 0.5 - x$ TPPi $0 < y = x - 0.5$
NTPP $y < 0.5 - x$ NTPPi $y < x - 0.5$
(1)

The resulting image of possible relations in RCC between intervals in \mathbb{R} , circular regions in \mathbb{R}^2 , and spheres in \mathbb{R}^3 is depicted in Fig. 1². Besides reflecting the CNG of RCC (neighboring relations in the CNG are neighboring regions, lines, or points), this image encodes two interesting aspects of evolution: (i) the

 $^{^{2}}$ This is different from Galton[11], since we normalize both the x- and y-axis metric with the sum of the radii to obtain a symmetric image.



Fig. 1. Combined image of topology, distance of boundaries, and size (cf. [11]).

implications of motion and scaling, and (ii) the dominance relationship between relations (e.g., EQ being a point indicates that it can hold for a time instant, whereas those relations being denoted by regions must hold for a time interval).

Considering the impact of evolution, a point in this figure denoting a particular relation of RCC moves along the x-axis when one of the spheres changes its size with respect to the size of the other sphere (i.e., due to scaling), whereas its movement along the y-axis is caused by motion of the centers (which in turn is either due to motion of an entire sphere, or scaling). For example, consider the black dot labelled $Holds(\phi_{rcc}(o, o'), NTPPi, s)$ in the sector NTPPi, denoting that o contains o': in terms of size and distance, this dot means that o is approximately twice the size of o' (cf. 0.67 on the x-axis), and that their centers are near each other. If o shrinks, the black dot moves along the x-axis to the left, until o can no longer fully contain o', leading to an overlap relation (represented by the dot moving from NTPPi into PO). During this scaling, at a single time instant the boundary of o touches the boundary of o', represented by the dot passing the line labeled TPPi (i. e., o' is a tangential proper part of o). If o shrinks even further, it will eventually be contained in o' (i.e., the dot will move into TPP). Now consider that the centers of o and o' coincide (i.e., their distance is zero): the same scaling event will then traverse EQ instead of TPPi, PO, and TPP.

We now define such a space representing relations as points for each of the employed positional relation calculi (distance of boundaries and size). To begin with, we discuss the integration of size, since the x-axis in Fig. 1 already expresses size relationships in terms of the ratio of interval radii r1/(r1+r2). The mapping to a qualitative size calculus is straightforward: a ratio below 0.5 corresponds to smaller (<), above 0.5 to larger (>) and one of exactly 0.5 to equal size (=).

Less obvious is the integration of the distance between boundaries. As a starting point, we informally define that two objects are very close whenever the boundaries meet, which is the case along the lines labeled EC, TPP, and TPPi, as well as at the point EQ. To both sides of these lines and around the point of topological equality, we define a region where the boundaries are still very close to each other (e.g., $10\%^3$ off in distance and size as used in Fig. 1). Since we must consistently encode the CNG (represented by the sequence VF-F-C-VC). to each side of VC a region C must follow, which itself neighbors to regions F. Finally, regions VF are positioned at the outermost and innermost sectors of the image, neighboring only to regions F. Considering PO in conjunction with VC, it becomes obvious why our metrics are normalized. Let o be much larger than o' $(r1 \gg r2)$ and o overlap with o': their boundaries certainly should be regarded to be very close to each other, since in comparison to the size of o the distance between their boundaries is quite small (analogous assumptions hold for $r1 \ll r2$). This means, that our image should be symmetric with respect to size equality (x = 0.5), which cannot be achieved using an unnormalized metric. In (2) below, we define the distance relation VC with respect to x = r1/(r1+r2) and y = d/2(r1 + r2). With analogous formalizations, C, F, and VF can be defined.

VC $0.45 < y \le 0.55 \lor 0.45 - x \le y \le 0.55 - x \lor x - 0.55 \le y \le x - 0.45$ (2)

Case study in the domain of road traffic management. We demonstrate the applicability of the integrated model by means of a hypothetic case study in the domain of road traffic management, which is made up of a situation evolution along various traffic entities, cf. Table 1. The entities are represented by traffic signs, and their spatial extent along the highway (direction from right to left) is indicated by surrounding boxes. The situation evolution comprises a traffic jam tj that starts growing due to capacity overload at a highway on-ramp onr in the middle of road works rwk. Shortly after, an accident acc occurs at the end of the traffic jam, which soon is contained within the further growing traffic jam. In order to reach the accident acc, an ambulance amb later passes through the traffic jam tj. In Table 1, the overall evolution of this situation is depicted as arrows representing evolution of relations in terms of their transitions in icons of the two-dimensional model introduced above. In order to represent traffic objects in our model, their occupied regions on a highway are modeled as intervals in \mathbb{R} . Next to each icon, Table 1 provides an informal description of the relation evolution between the entities. Summing up the case study, we have illustrated our approach by applying it to a scenario involving various different aspects of evolution: (i) scaling in comparison to stationary objects with and without

³ This measure has simply been chosen due to ease of presentation. It has neither been determined nor tested using cognitive studies.



 Table 1. Case study of situation evolution in terms of relation transitions.

leading to relation transitions, (ii) non-evolution between two stationary, nonscaling entities, and (iii) motion of a non-scaling entity with respect to a scaling, and to a stationary, non-scaling one. The inter-calculi dependencies of Fig. 1 are extracted as qualification constraints into an ontology in the next section.

4 An Ontology of Inter-Calculi Dependencies

Since we focus on the dynamic behavior of spatial systems, we express the inter-calculi dependencies summarized above in Fig. 1 in an ontology on the basis of the Situation Calculus [22] providing explicit support for modeling change between states in the form of occurrents. Change in the Situation Calclus is manifested in the properties of entities and the relations between them (e.g., a traffic jam's position can change, or its distance relation to an accident). In the terminology of the Situation Calculus, entities are continuants $O = \{o_1, o_2, \ldots, o_n\}$, whereas their properties and relations to other entities in a particular situation are referred to as *fluents* $\Phi = \{\phi_1(o_1), \phi_2(o_2), \dots, \phi_n(o_n)\}.$ We use in accordance with [3] relational fluents ϕ_r relating two continuants to each other using denotation sets $\Gamma = \{\gamma_1, \gamma_2, \ldots, \gamma_n\}$ and a ternary predicate Holds denoting that a fluent holds a particular value in a particular situation: for instance, $Holds(phi_{rcc8}(o, o'), EQ, s)$ describes that the objects o and o' are topologically equal in situation s. Changed fluents are the result of *occurrents* $\Theta = \{\theta_1(o_1, \theta_2(o_2, \dots, \theta_n(o_n)) | 3].$ Qualification constraints for occurrents can be defined using axioms of the form $Poss(\theta(\mathbf{o}), s) \equiv Holds(\phi(\mathbf{o}_1), \gamma, s)$, denoting that θ is possible in situation s, when a fluent ϕ of entities o_1 holds a particular value γ in s. For example, being of small size might be a precondition for growing, as stated by $(\forall o \in O)(\forall s \in S)Poss(grow(o), s) \equiv Holds(size(o), small, s).$

Utilizing the Situation Calculus, we define qualification constraints for relation transitions (occurrents) on the basis of relational fluents, cf. Def. 1. **Definition 1 (Inter-calculi qualification constraint).** In accordance with [3] let transitions between relations be occurrents $tran(\gamma, o, o')$, meaning that o and o' transition to the relation γ . An inter-calculi qualification constraint can then be formulated in the Situation Calculus as an action precondition axiom [22] of the syntactic form given in (3), meaning that a transition to γ_1 is possible, if a relation γ_2 of another spatial calculus currently holds between o and o'.

 $(\forall o, o' \in O)(\forall s \in S)Poss(tran(\gamma_1, o, o'), s) \equiv Holds(\phi_{spatial}(o, o'), \gamma_2, s) \quad (3)$

In Ex. 1, we provide a sample inter-calculi transition qualification constraint that formalizes the preconditions of the transition between DC and EC in terms of the states of relational fluents defining qualitative size and distance relationships.

Example 1. A transition from DC to EC in RCC is possible, if (trivially) DC from RCC holds, from a distance point-of-view VC holds, and from a size point-of-view any relation holds (summarized by the light-gray region VC that borders EC and spans all size relations in Fig. 1).

$$(\forall o, o' \in O)(\forall s \in S)Poss(tran(EC, o, o'), s) \equiv Holds(\phi_{rcc8}(o, o'), DC, s)$$

$$\wedge Holds(\phi_{dist}(o, o'), VC, s) \wedge Holds(\phi_{size}, \gamma_1, s) \text{where } \gamma_1 \in \{<, =, >\}$$
(4)

As a proof-of-concept, we implemented the conceptual neighborhood structure of RCC, spatial distance of boundaries, and size, as well as the abovedefined constraints in SWI-Prolog and used the FSA planner [16] implementing GOLOG (Reiter's Situation Calculus programming language [22]) to synthesize sequential plans comprising the necessary relation transitions in order to reach a future goal situation from a current one. The lessons learned from this prototypical implementation and directions for further work are summarized below.

5 Critical Discussion and Further Work

Synthesized plans reflect commonsense understanding of evolution. The synthesized plans, without inter-calculi qualification constraints, reflect some implementation-dependent choice of the planner between independent transitions being possible at the same time (e.g., in our test runs, the order of transitions in the plan corresponded with the order of relations in the initial situation definition). Considering the additional inter-calculi qualification constraints, these transitions are no longer independent and, hence, the synthesized plans are consistent with commonsense understanding of the evolution of entities.

Generalization in terms of calculi and spatial primitives. Existing topological and positional calculi (e.g., Egenhofer's approach [7]) can be integrated into the model by defining for each relation of the calculus a mapping to the x- and y-coordinate measures of our model. For example, the relation inside modeled as 4-intersection $\begin{pmatrix} \neg \emptyset & \emptyset \\ \neg \emptyset & \emptyset \end{pmatrix}$ describes a relation between two objects o and o', where the intersection of the interiors of o and o' is not empty, the intersection of the interior of o' is not empty, whereas the

intersection of the interior of o and the boundary of o', as well as the intersection of their boundaries are empty. In terms of our model, for such a relation the distance between the centroids of o and o' must be smaller than the difference between their radii (d < r2 - r1), hence the following must hold true: d/2(r1 + r2) < 0.5 - r1/(r1 + r2) (i.e., **inside** is NTPP of RCC). In order to integrate additional spatial aspects not being representable with the spatial primitives employed above (e.g., orientation of entities towards each other), a generalization (e.g., in terms of higher-dimensional images) of the presented abstraction in terms of radii and center distance of spatial primitives is still necessary (e.g., considering orientation vectors). Likewise, in order to support the multitude of different spatial primitives found especially in GIS (e.g., regions, lines, points, as well as fuzzy approaches with broad boundaries) going beyond the intervals, regions, and spheres utilized above, metrics for comparing spatial primitives of different sorts must be defined (e.g., a line passing a region [7]).

Encoding of the ontology with Semantic Web standards. Since current Semantic Web standards, in particular OWL 2, formalize ontologies using a decidable fragment of first-order logic, an interesting further direction is to define a mapping of the ontology excerpt expressed in terms of the Situation Calculus into the concepts of OWL 2. For this, it can be based on prior work in terms of description logic rules [19] integrating rules and OWL. As a result, an integration with Semantic-Web-based GIS would be an interesting option.

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