Feature extraction through LOCOCODE
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Sepp Hochreiter
Fakultät für Informatik
Technische Universität München
80290 München, Germany
hochreit@informatik.tu-muenchen.de
http://www7.informatik.tu-muenchen.de/~hochreit

Jürgen Schmidhuber
IDSIA
Corso Elvezia 36
6900 Lugano, Switzerland
juergen@idsia.ch
http://www.idsia.ch/~juergen

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Abstract

"*Lo*w-*co*mplexity *co*ding and *de*coding" (LOCOCODE) is a novel approach to sensory coding and unsupervised learning. Unlike previous methods it explicitly takes into account the information-theoretic complexity of the code generator: it computes *lococodes* that (1) convey information about the input data and (2) can be computed and decoded by low-complexity mappings. We implement LOCOCODE by training autoassociators with *Flat Minimum Search*, a recent, general method for discovering low-complexity neural nets. It turns out that this approach can unmix an unknown number of independent data sources by extracting a minimal number of low-complexity features necessary for representing the data. Experiments show: unlike codes obtained with standard autoencoders, lococodes are based on feature detectors, never unstructured, usually sparse, sometimes factorial or local (depending on statistical properties of the data). Although LOCOCODE is not explicitly designed to enforce sparse or factorial codes, it extracts optimal codes for difficult versions of the "bars" benchmark problem, whereas ICA and PCA do not. It also produces familiar, biologically plausible feature detectors when applied to real world images. As a preprocessor for a vowel recognition benchmark problem it sets the stage for excellent classification performance. Our results reveil an interesting, previously ignored connection between two important fields: regularizer research, and ICA-related research.

1 INTRODUCTION

What is the goal of sensory coding (Field 1994)? There is no generally agreed-upon answer yet. Several information-theoretic objective functions (OFs) have been proposed to evaluate the quality of sensory codes. Most OFs focus on statistical properties of the code components (such as mutual information) — we refer to them as code component-oriented OFs, or COCOFs. Some COCOFs explicitly favor near-factorial, minimally redundant codes of the input data (see, e.g., Watanabe 1985, Barlow et al. 1989, Linsker 1988, Schmidhuber 1992, Schmidhuber and Prelinger 1993, Schraudolph and Sejnowski 1993, Redlich 1993, Deco and Parra 1994). Such codes can be advantageous for (1) data compression, (2) speeding up subsequent gradient descent learning (e.g., Becker 1991), (3) simplifying subsequent Bayes classifiers (e.g., Schmidhuber et al. 1996). Other approaches favor local codes, e.g., Rumelhart and Zipser (1986), Barrow (1987), Kohonen (1988). They can help to achieve (1) minimal crosstalk, (2) subsequent gradient descent speed-ups, (3) facilitation of post training analysis, (4) simultaneous representation of different data items. Recently there also has been much work on COCOFs encouraging biologically plausible sparse distributed codes, e.g., Field (1987), Barlow (1983), Mozer (1991), Földiák (1990), Földiák

and Young (1995), Palm (1992), Zemel and Hinton (1994), Field (1994), Saund (1994), Dayan and Zemel (1995), Li (1995), Olshausen and Field (1996), Zemel (1993), Hinton and Ghahramani (1997). Sparse codes share certain advantages of both local and dense codes.

But what about coding costs? COCOFs express desirable properties of the code itself, while neglecting the costs of constructing the code from the data. For instance, coding input data without redundancy may be very expensive in terms of information bits required to describe the code-generating network, which may need many finely tuned free parameters.

A previous argument for ignoring coding costs (e.g., Zemel 1993, Zemel and Hinton 1994, Hinton and Zemel 1994), based on the principle of minimum description length (MDL, e.g., Solomonoff 1964, Wallace and Boulton 1968, Rissanen 1978), focuses on hypothetical costs of transmitting the data from some sender to a receiver — how many bits are necessary to enable the receiver to reconstruct the data? It goes more or less like this: *"Total transmission cost is the number of bits required to describe (1) the data's code, (2) the reconstruction error and (3) the decoding procedure. Since all input exemplars are encoded/decoded by the same mapping, the coding/decoding costs are negligible because they occur only once."*

We doubt, however, that sensory coding's sole objective should be to transform data into a compact code that is cheaply transmittable across some ideal, abstract channel. In fact, the most compact code of the possible environmental inputs would be the "true" probabilistic causal model corresponding to our universe's most basic physical laws. Generating this code and using it for dealing with everyday events, however, would be far too costly. We believe that one of sensory coding's objectives should be to reduce the cost of code *generation* through data transformations in *existing* channels (e.g., synapses etc.)[1]. Without denying the usefulness of certain COCOFs, we postulate that an important scarce resource is the bits required to describe the mappings that generate and process the codes — after all, it is these mappings that need to be implemented, given some limited hardware.

Lococodes. For such reasons we shift the point of view and focus on the information-theoretic costs of code-generation. We will present a novel approach to unsupervised learning called "*lo*w-*co*mplexity *co*ding and *de*coding" (LOCOCODE — see also Hochreiter and Schmidhuber, 1997b, 1997c). Without assuming particular goals such as data compression, simplifying subsequent classification, etc., but in the MDL spirit, LOCOCODE generates so-called *lococodes* that (1) convey information about the input data, (2) can be computed from the data by a low-complexity mapping (LCM), (3) can be decoded by a LCM. By minimizing coding/decoding costs LOCOCODE will yield efficient, robust, noise-tolerant mappings for processing inputs and codes.

Lococodes through FMS. To implement LOCOCODE we apply *Flat Minimum Search* (FMS, Hochreiter and Schmidhuber 1997a) to an autoassociator (AA) whose hidden layer activations represent the code. FMS is a general, gradient-based method for finding networks that can be described with few bits of information.

Coding each input via few simple basis functions. A basis function is the function determining the activation of a code component in response to a given input. The analysis in Section 3 will show that LOCOCODE tries to reproduce the current input by using as few code components as possible, each computed by a separate low-complexity basis function (implementable, e.g., by a subnetwork with few low-precision weights).

This reflects a basic assumption, namely, that the true input "causes" (e.g., Hinton et al. 1995, Dayan and Zemel 1995, Ghahramani 1995) are indeed few and simple. Training sets whose elements are all describable by few features will result in *sparse* codes, where sparseness does not necessarily mean that there are "few active code components" but that "few code components contribute to reproducing the input" (this makes a difference in the nonlinear case).

We will see that LOCOCODE encourages noise-tolerant feature detectors reminiscent of those observed in the mammalian visual cortex. Inputs that are mixtures of a few regular features, such as edges in images, can be described well in a sparse fashion (only code components corresponding to present features contribute to coding the input). In contrast to previous approaches, however, sparseness is not viewed as an *a priori* good thing, and is not enforced explicitely, but only if the

[1] Note that the mammalian visual cortex rarely just transmits data without also transforming it.

input data indeed can be described naturally in a sparse fashion. Some lococodes are not only sparse but also factorial, depending on whether the input is decomposable into factorial features. Likewise lococodes may deviate from sparseness towards locality if each input exhibits a single characteristic feature. Then the code will not be factorial (because knowledge of the component representing the characteristic feature implies knowledge of all others), but it will still be natural because it represents the true cause in a fashion that makes reconstruction (and other types of further processing) simple.

Outline. An FMS-review will follow in Section 2. Section 3 will analyze the beneficial effects of FMS' error terms in the context of autoencoding. The remainder of our paper will be devoted to empirical justifications of LOCOCODE. Experiments in Section 4.2 will show that all three "good" kinds of code discussed in previous work (namely local, sparse, factorial) can be natural lococodes. In Section 4.3 LOCOCODE will extract optimal sparse codes reflecting the independent features of random horizontal and vertical (noisy) bars, while ICA and PCA won't. In Section 4.4 LOCOCODE will generate plausible sparse codes (based on well-known on-center-off-surround and other appropriate feature detectors) of real world images. Section 4.5 will finally use LOCOCODE as a preprocessor for a standard, overfitting back-propagation (BP) speech data classifier. Surprisingly, this combination achieves excellent generalization performance. We conclude that the speech data's lococode already conveys the "essential", noise-free information, already in a form useful for further processing and classification. Section 5 will discuss our findings.

2 FLAT MINIMUM SEARCH: REVIEW

FMS Overview. FMS is a general method for finding low complexity-networks with high generalization capability. FMS finds a large region in weight space such that each weight vector from that region has *similar* small error. Such regions are called "flat minima". In MDL terminology, few bits of information are required to pick a weight vector in a "flat" minimum (corresponding to a low complexity-network) — the weights may be given with low precision. In contrast, weights in a "sharp" minimum require a high-precision specification. As a natural by-product of net complexity reduction, FMS automatically prunes weights (by setting them to zero) and units (e.g., by giving them a strong bias), and reduces output sensitivity with respect to remaining weights and units. Previous FMS applications focused on supervised learning (Hochreiter and Schmidhuber 1995, 1997a): FMS led to better stock market prediction results than "weight decay" and "optimal brain surgeon" (Hassibi and Stork 1993). In this paper, however, we will use it for unsupervised coding only.

Architecture. We use a 3-layer feedforward net. Each layer is fully connected to the next layer. Let O, H, I denote index sets for output, hidden, input units, respectively. Let $|.|$ denote the number of elements in a set. For $l \in O \cup H$, the activation y^l of unit l is $y^l = f(s_l)$, where $s_l = \sum_m w_{lm} y^m$ is the net input of unit l ($m \in H$ for $l \in O$ and $m \in I$ for $l \in H$), w_{lm} denotes the weight on the connection from unit m to unit l, f denotes the activation function, and for $m \in I$, y^m denotes the m-th component of an input vector. $W = |(O \times H) \cup (H \times I)|$ is the number of weights.

Algorithm. FMS' objective function E features an unconventional error term:

$$B = \sum_{i,j \in O \times H \cup H \times I} \log \sum_{k \in O} \left(\frac{\partial y^k}{\partial w_{ij}} \right)^2 + W \log \sum_{k \in O} \left(\sum_{i,j \in O \times H \cup H \times I} \frac{\left| \frac{\partial y^k}{\partial w_{ij}} \right|}{\sqrt{\sum_{k \in O} \left(\frac{\partial y^k}{\partial w_{ij}} \right)^2}} \right)^2 .$$

$E = E_q + \lambda B$ is minimized by gradient descent, where E_q is the training set mean squared error (MSE), and λ a positive "regularizer" scaling B's influence. Defining λ corresponds to choosing a tolerable error level (there is no *a priori* "optimal" way of doing so). B measures the weight precision (number of bits needed to describe all weights in the net). Reducing B without increasing E_q means removing weight precision without increasing MSE. Given a constant number of output

units, all of this can be done efficiently, namely, with standard BP's order of computational complexity. For details see Hochreiter and Schmidhuber's article (1997a) or their home pages. For even more general attempts at reducing net complexity see Schmidhuber (1997a).

3 EFFECTS OF THE ADDITIONAL TERM B

Where does B come from? To discover flat minima FMS searches for large axis-aligned hypercuboids (boxes) in weight space such that weight vectors within the box yield similar network behavior. Boxes satisfy two flatness conditions, FC1 and FC2. FC1 enforces "tolerable" output variation in response to weight vector perturbations, i.e., near-flatness of the error surface around the current weight vector (in all weight space directions). Among the boxes satisfying FC1, FC2 selects a unique one with minimal net output variance. B is the negative logarithm of this box's volume (ignoring constant terms that have no effect on the gradient descent algorithm). Hence B is the number of bits (save a constant) required to describe the current net function, which does not change significantly by changing weights within the box. The box edge length determines the required weight precision. See Hochreiter and Schmidhuber (1997a) for details of B's derivation.

3.1 First term of B: sparseness and simple basis functions preferred.

Simple basis functions. The term

$$T1 := \sum_{i,j \in O \times H \cup H \times I} \log \sum_{k \in O} \left(\frac{\partial y^k}{\partial w_{ij}} \right)^2$$

reduces output sensitivity with respect to weights (and, therefore, units). $T1$ is responsible for pruning weights (and, therefore, units). $T1$ makes (1) unit activations decrease to zero in proportion to their fan-outs, (2) first-order derivatives of activation functions decrease to zero in proportion to their fan-ins, and (3) the influence of units on the output decrease to zero in proportion to the unit's fan-in. For a detailed analysis see Hochreiter and Schmidhuber (1997a). $T1$ is the reason why low-complexity (or simple) basis functions are preferred.

Sparseness. Point (1) above favors sparse hidden unit activations; point (2) favors non-informative hidden unit activations hardly affected by small input changes. Point (3) favors sparse hidden unit activations in the sense that "few hidden units contribute to producing the output". In particular, with sigmoid hidden units active in $[0, 1]$, near-zero activations are preferred.

3.2 Second term: few, separated, common basis functions preferred.

The term

$$T2 := W \log \sum_{k \in O} \left(\sum_{i,j \in O \times H \cup H \times I} \frac{\left| \frac{\partial y^k}{\partial w_{ij}} \right|}{\sqrt{\sum_{k \in O} \left(\frac{\partial y^k}{\partial w_{ij}} \right)^2}} \right)^2 ,$$

punishes units with similar influence on the output. We reformulate it:

$$T2 = W \log \left(\sum_{i,j \in O \times H \cup H \times I} \sum_{u,v \in O \times H \cup H \times I} \frac{\sum_{k \in O} \left| \frac{\partial y^k}{\partial w_{ij}} \right| \left| \frac{\partial y^k}{\partial w_{uv}} \right|}{\sqrt{\sum_{k \in O} \left(\frac{\partial y^k}{\partial w_{ij}} \right)^2} \sqrt{\sum_{k \in O} \left(\frac{\partial y^k}{\partial w_{uv}} \right)^2}} \right) .$$

Using

$$\frac{\partial y^k}{\partial w_{ij}} = \frac{\partial y^k}{\partial y^i} \frac{\partial y^i}{\partial w_{ij}},$$

this can be rewritten as

$$T2 \quad = \quad W \log \left(\sum_{i,j \in O \times H \cup H \times I} \quad \sum_{u,v \in O \times H \cup H \times I} \frac{\sum_{k \in O} \left| \frac{\partial y^k}{\partial y^i} \right| \left| \frac{\partial y^k}{\partial y^u} \right|}{\sqrt{\sum_{k \in O} \left(\frac{\partial y^k}{\partial y^i} \right)^2} \sqrt{\sum_{k \in O} \left(\frac{\partial y^k}{\partial y^u} \right)^2}} \right) .$$

For $i, j \in O \times H$

$$\frac{\left| \frac{\partial y^k}{\partial y^i} \right|}{\sqrt{\sum_{k \in O} \left(\frac{\partial y^k}{\partial y^i} \right)^2}} = 1$$

holds. We obtain

$$T2 \quad = \quad W \log \left(|O| \ |O \times H|^2 + |I|^2 \sum_{k \in O} \sum_{i \in H} \sum_{u \in H} \frac{\left| \frac{\partial y^k}{\partial y^i} \right| \left| \frac{\partial y^k}{\partial y^u} \right|}{\sqrt{\sum_{k \in O} \left(\frac{\partial y^k}{\partial y^i} \right)^2} \sqrt{\sum_{k \in O} \left(\frac{\partial y^k}{\partial y^u} \right)^2}} \right) .$$

We observe: (1) an output unit that is very sensitive with respect to two given hidden units will heavily contribute to $T2$ (compare the numerator in the last term in the brackets of $T2$). (2) This large contribution can be reduced by making both hidden units have large impact on other output units (see denominator in the last term in the brackets of $T2$).

Choice of basis functions. FMS tries to figure out a way of using (1) as few basis functions as possible for each output unit (this leads to separation of basis functions), while simultaneously (2) using the same basis functions for as many output units as possible (common basis functions).

SPECIAL CASE: LINEAR OUTPUT ACTIVATION.

Since our targets will usually be in the linear range of a sigmoid output activation function, let us consider the linear case in more detail. Suppose all output units k use the same linear activation function $f_k(x) = Cx$ (where C is a real-valued constant). Then $\frac{\partial y^k}{\partial y^i} = Cw_{ki}$ for hidden unit i. We obtain

$$T2 \quad = \quad W \log \left(|O| \ |O \times H|^2 + |I|^2 \sum_{i \in H} \sum_{u \in H} \frac{\sum_{k \in O} |w_{ki}| \ |w_{ku}|}{\|W_i\| \ \ \|W_u\|} \right) ,$$

where W_i denotes the outgoing weight vector of unit i with $[W_i]_k := w_{ki}$, $\|.\|$ the Euclidean vector norm $\|x\| = \sqrt{\sum_i x_i^2}$, and $[.]_k$ the kth component of a vector.

Few basis functions preferred. We observe that hidden units whose outgoing weight vectors have near-zero weights yield small contributions to $T2$, that is, the number of basis functions will get minimized.

Common basis functions preferred. Outgoing weight vectors of hidden units are encouraged to have a large effect on the output (see denominator in the last term in the brackets of $T2$). This implies preference of basis functions that can be used for generating many or all output components.

Basis function separation — few relevant basis functions per output unit. On the other hand, two hidden units whose outgoing weight vectors do not solely consist of near-zero weights are encouraged to influence the output in different ways by not representing the same input feature (see numerator in the last term in the brackets of $T2$). In fact, FMS punishes not only outgoing weight vectors with same or opposite directions but also vectors obtained by flipping the signs of the weights (multiple reflections from hyperplanes trough the origin and orthogonal to one axis). Hence two units performing redundant tasks, such as both activating some ouput unit, or one activating it and the other de-activating it, will cause large contributions to $T2$. This encourages separation of basis functions and use of few basis functions per output unit.

3.3 Low-Complexity Autoassociators

Given some data set, FMS can be used to find a low-complexity autoassociator (AA) whose hidden layer activations code the individual training exemplars. The AA can be split into two modules: one for coding, one for decoding.

Previous autoassociators (AAs). Backprop-trained AAs *without* a narrow hidden bottleneck ("bottleneck" refers to a hidden layer containing fewer units than other layers) typically produce redundant, continuous-valued codes and unstructured weight patterns. Baldi and Hornik (1989) studied linear AAs *with* a hidden layer bottleneck and found that they produce codes that are orthogonal projections onto the subspace spanned by the first principal eigenvectors of a covariance matrix associated with the training patterns. They showed that the mean squared error (MSE) surface has an unique minimum. Nonlinear codes have been obtained by nonlinear bottleneck AAs with more than 3 (e.g., 5) layers, e.g., Kramer (1991), Oja (1991) or DeMers and Cottrell (1993). None of these methods produces sparse, factorial or local codes — instead they produce first principal components or their nonlinear equivalents ("principal manifolds"). We will see that FMS-based AAs yield quite different results.

FMS-based AAs. According to subsections 3.1 and 3.2, because of the low-complexity *coding* aspect the codes tend to (C1) be binary for sigmoid units active in [0,1] (values near 0 or 1 can easily be generated with low-precision weights), (C2) require few code components or hidden units (HUs) (few units and weights needed for presenting the input), and (C3) use simple basic functions (with low-precision weights). Because of the low-complexity *de*coding part, codes also tend to (D1) have many HUs near zero (then the outgoing weights may be unprecise and few units and weights are needed for decoding) and, therefore, be sparsely (or even locally) distributed, (D2) have code components conveying information useful for generating as many output activations as possible (all HUs are equally important — fewer weights are needed). (C1), (C2) and (D2) encourage minimally redundant, binary codes. (C3), (D1) and (D2), however, encourage sparse distributed (local) codes. (C1) – (C3) and (D1) – (D2) lead to codes with simply computable code components (C1, C3) that convey a lot of information (D2), and with as few active code components as possible (C2, D1). *Collectively this makes code components represent simple input features.*

4 EXPERIMENTS

Outline. Section 4.1 provides an overview over the experimental conditions. Section 4.2 uses simple artificial tasks to show how various lococode types (factorial, local, sparse, feature detector-based) depend on input/output properties. The visual coding experiments are divided into two sections: Section 4.3 deals with artificial bars, Section 4.4 with real world images. In Section 4.3 the "true" causes of the input data are known, and we show that LOCOCODE learns to represent them optimally (PCA and ICA do not). In Section 4.4 it generates plausible feature detectors. Finally, in Section 4.5 LOCOCODE is used as a preprocessor for speech data fed into standard backpropagation classifier. This provokes significant performance improvement.

4.1 Experimental Conditions

In all our experiments we associate input data with itself, using an FMS-trained 3-layer autoassociator (AA) with semilinear activation function in the hidden layer. Unless stated otherwise we use 700,000 training exemplars, sigmoid hidden units (HUs) active in [0,1], sigmoid output units active in [-1,1], noninput units with an additional bias input, normal weights initialized in $[-0.1, 0.1]$, bias hidden weights with -1.0, λ with 0.5.

Parameters and other details.

- learning rate: conventional learning rate for error term E (just like backprop's).
- λ: a positive "regularizer" (hyperparameter) scaling B's influence. λ is computed heuristically as described by Hochreiter and Schmidhuber (1997a).

- $\Delta\lambda$: a value used for updating λ during learning. It represents the absolute change of λ after each epoch.
- E_{tol}: the tolerable mean squared error (MSE) on the training set. It is used for dynamically computing λ, and for deciding when to switch phases in 2-phase learning.
- 2-phase learning speeds up the algorithm: phase 1 is conventional backprop, phase 2 is FMS. We start with phase 1 and switch to phase 2 once $E_a < E_{tol}$, where E_a is the average epoch error. We switch back to phase 1 once $E_a > \gamma\ E_{tol}$. We finish in phase 2. The experimental sections will indicate 2-phase learning by mentioning values of γ.
- Pruning of weights and units: we judge a weight w_{ij} as being pruned if its required precision (δw_{ij} in Hochreiter and Schmidhuber's 1997a article) for each input is 100 times lower (corresponding to 2 decimal digits) than the highest precision of the other weights for the same input. A unit is considered pruned if all incoming weights are pruned except for the bias weight, or if all outgoing weights are pruned.

For more details see Hochreiter and Schmidhuber (1997a) or their home pages.

Comparison. In sections 4.3 and 4.4 we compare LOCOCODE to "independent component analysis" (ICA, e.g., Cardoso and Souloumiac 1993, Molgedey and Schuster 1994, Bell and Sejnowski 1995, Amari et al. 1996) and "pricipal component analysis" (PCA, e.g., Oja 1989). ICA is realized by Cardoso's (1993) JADE (Joint Approximate Diagonalization of Eigen-matrices) algorithm (we used the Matlab JADE version obtained via FTP from `sig.enst.fr`). JADE is based on whitening and subsequent joint diagonalization of 4th-order cumulant matrices. For PCA and ICA, 1,000 (3,000) training exemplars are used in case of 5×5 (7×7) input fields.

Information content? To measure the information conveyed by the various codes obtained in sections 4.3 and 4.4 we train a standard backprop net on the training set used for code generation. Its inputs are the code components; its task is to reconstruct the original input (for all tasks except for "noisy bars" the original input is scaled such that all input components are in $[-1.0, 1.0]$). The net has as many biased sigmoid hidden units active in $[0, 1]$ as there are biased sigmoid output units active in $[-1, 1]$. We train it for 5,000 epochs without caring for overfitting. The training set consists of 500 fixed exemplars in the case of 5×5 input fields and of 5000 in the case of 7×7 input fields. The test set consists of 500 off-training set exemplars (in the case of real world images we use a separate test image). The average MSE on the test set is used to determine the reconstruction error.

4.2 EXPERIMENT 1: local, sparse, factorial codes — feature detectors

Task. The data consists of 8 input vectors with 8 components each. The i-th component of the i-th vector is 0.8. The other components are 0.2. The sigmoid hidden units (HUs) are active in [0,1]. Code units and output units have an additional bias input. Code units are initialized with a negative bias of -2.0.

Experiment 1.1: uniformly distributed inputs, 500,000 training examples. Each input vector has probability $\frac{1}{8}$ of being chosen next. *Parameters:* learning rate: 0.1, the "tolerable error" $E_{tol} = 0.1$, $\Delta\lambda = 1.0$, $\gamma = 2.0$. *Architecture:* (8-5-8) (8 input units, 5 HUs, 8 output units).

Results: factorial codes. In 7 out of 10 trials, FMS effectively pruned 2 HUs, and produced a *factorial binary code* with statistically independent code components. In 2 trials FMS pruned 2 HUs and produced an almost binary code — with one trinary unit taking on values of 0.0, 0.5, 1.0. In one trial FMS produced a binary code with only one HU being pruned away. Obviously, under certain constraints on the input data, FMS has a strong tendency towards the compact, nonredundant codes advocated by numerous researchers.

Experiment 1.2: like Experiment 1.1, but with architecture (8-8-8), 200,000 training examples and input values in $\{0.0, 1.0\}$ (as opposed to input values in $\{0.2, 0.8\}$). All other parameters are like in Experiment 1.1. We use more HUs than in Experiment 1.1, to make clear that in this case fewer units are pruned.

Results: local codes. 10 trials were conducted. FMS always produced a binary code. In 7 trials, only 1 HU was pruned, in the remaining trials 2 HUs. Unlike with standard BP, *almost all inputs almost always were coded in an entirely local manner*, i.e., only one HU was switched on, the others switched off. Recall that local codes were also advocated by many researchers – but they are precisely "the opposite" of the factorial codes from the previous experiment. How can LOCOCODE justify such different codes? How to explain this apparent discrepancy?

Explanation. The reason is: with the different input representation, the additional HUs do not necessarily result in much more additional complexity of the mappings for coding and decoding. The zero-valued inputs allow for low weight precision (low coding complexity) for connections leading to HUs (similarly for connections leading to output units). In contrast to Experiment 1.1 it is possible to describe the i-th possible input by the following feature: "the i-th input component does not equal zero". It can be implemented by a low-complexity basis function. This contrasts the features in Experiment 1.1, where there are only 5 hidden units and no zero input components: there it is better to code with as few code components as possible, which yields a factorial code.

Experiment 1.3: like Experiment 1.2, but architecture (1-8-1) and *one-dimensional* inputs: 0.05, 0.1, 0.15, 0.2, 0.8, 0.85, 0.9, 0.95. *Parameters:* learning rate: 0.1, $E_{tol} = 0.00004$, $\Delta\lambda = 1.0$, $\gamma = 2.0$.

Results: feature detectors. 10 trials were conducted. FMS always produced the following code: one binary HU making a distinction between input values less than 0.5 and input values greater than 0.5, 2 HUs with continuous values, one of which is zero (or one) whenever the binary unit is on, while the other is zero (one) otherwise. All remaining HUs adopt constant values of either 1.0 or 0.0, thus being essentially pruned away. The binary unit serves as a binary *feature detector*, grouping the inputs into 2 classes.

Lococode recognizes the causes. The data is generated as follows: (1) first choose with uniform probability a value from $\{0.0, 0.75\}$; then (2) choose one from $\{0.05, 0.1, 0.15, 0.2\}$; then (3) add the two values. The first cause of the data is recognized perfectly but the second is divided among two code components, due to the non-linearity of the output unit: adding to 0 is different from adding to 0.75 (consider the first order derivatives).

Experiment 1.4: nonuniformly distributed inputs. Input vectors like in Experiment 1.1, but occurring with probabilities $\frac{1}{4}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}$. *Parameters:* learning rate: 0.005, $E_{tol} = 0.01$, $\Delta\lambda = 1.0$, $\gamma = 2.0$. *Architecture:* (8-5-8).

Results: sparse codes. In 4 out of 10 trials, FMS found a binary code (no HUs pruned). In 3 trials: a binary code with one HU pruned. In one trial: a code with one HU removed, and a trinary unit adopting values of 0.0, 0.5, 1.0. In 2 trials: a code with one pruned HU and 2 trinary HUs. Obviously, with this set-up, FMS prefers codes known as *sparse distributed representations*. Inputs with higher probability are coded by fewer active code components than inputs with lower probability. Typically, inputs with probability $\frac{1}{4}$ lead to one active code component, inputs with probability $\frac{1}{8}$ to two, and others to three.

Explanation. Why is the result different from Experiment 1.1's? To achieve equal error contributions to all inputs, the weights for coding/decoding highly probable inputs have to be given with higher precision than the weights for coding/decoding inputs with low probability: the input distribution from Experiment 1.1 will result in a more complex network. The next experiment will make this effect even more pronounced.

Experiment 1.5: like Experiment 1.4, but with architecture (8-8-8).

Results: sparse codes. In 10 trials, FMS always produced binary codes. In 2 trials only 1 HU was pruned. In 1 trial 3 units were pruned. In 7 trials 2 units were pruned. Unlike with standard BP, *almost all inputs almost always were coded in a sparse, distributed manner:* typically, 2 HUs were switched on, the others switched off, and most HUs responded to exactly 2 different input patterns. The mean probability of a unit being switched on was 0.28, and the probabilities of different HUs being switched on tended to be equal.

Table 1 provides an overview over Experiments 1.1 — 1.5.

Conclusion. FMS always finds codes quite different from standard BP's rather unstructured ones. It tends to discover and represent the underlying causes. Usually the resulting lococode is

Exp.	input coding	input values	input distribution	architecture	code components	result
1.1	local	0.2, 0.8	uniform	8-5-8	3	factorial code
1.2	local	0.0, 1.0	uniform	8-8-8	7	local code
1.3	dense	0.05, 0.1, 0.15, 0.2, 0.8, 0.85, 0.9, 0.95	uniform	1-8-1	3	feature detectors
1.4	local	0.2, 0.8	$\frac{1}{4}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}$	8-5-8	4	sparse code
1.5	local	0.2, 0.8	$\frac{1}{4}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}$	8-8-8	6	sparse code

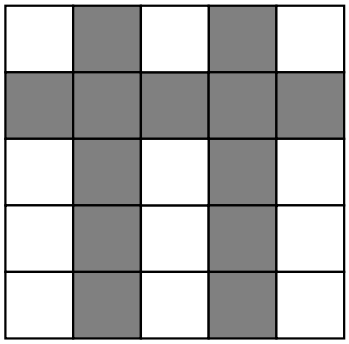
Table 1: *Overview over experiments 1.1 – 1.5: type of input coding, possible values of input components, distribution of the 8 input vectors, architecture in the form "input-hidden-output" units, nature of the resulting lococode (which mainly depends on the nature of the input data).*

sparse and based on informative feature detectors. Depending on properties of the data it may become factorial or local. This suggests that LOCOCODE may represent a general principle of unsupervised learning subsuming previous, COCOF-based approaches.

Feature-based lococodes automatically take into account input/output properties (binary?, local?, input probabilities?, noise?, number of zero input components?).

4.3 EXPERIMENT 2: Independent Bars

Task 2.1 — adapted from Dayan and Zemel (1995), see also Földiák (1990), Zemel (1993), Saund (1995), but more difficult (compare M. Baumgartner's 1996 diploma thesis). The input is a 5×5 pixel grid with horizontal and vertical bars at random, independent positions. See Figure 1 for an example. The task is to extract the independent features (the bars). According to Dayan and



-0.5	**+0.5**	**-0.5**	**+0.5**	**-0.5**
+0.5	**+0.5**	**+0.5**	**+0.5**	**+0.5**
-0.5	**+0.5**	**-0.5**	**+0.5**	**-0.5**
-0.5	**+0.5**	**-0.5**	**+0.5**	**-0.5**
-0.5	**+0.5**	**-0.5**	**+0.5**	**-0.5**

Figure 1: *Task 2.1: example of partly overlapping bars. The 2nd and the 4th vertical bar and the 2nd horizontal bar are switched on simultaneously. Left: the corresponding input values.*

Zemel (1995), even a simpler variant (where vertical and horizontal bars may not be mixed in the same input) is not trivial:

> "Although it might seem like a toy problem, the 5×5 bar task with only 10 hidden units turns out to be quite hard for all the algorithms we discuss. The coding cost of

making an error in one bar goes up linearly with the size of the grid, so at least one aspect of the problem gets *easier* with large grids."

We will see that even difficult variants of this task are not hard for LOCOCODE.

Training and testing. Each of the 10 possible bars appears with probability $\frac{1}{5}$. In contrast to Dayan and Zemel's set-up (1995) we allow for bar type mixing. This makes the task harder (Dayan and Zemel 1995, p. 570). To test LOCOCODE's ability to reduce redundancy, we use many more HUs (namely 25) than the required minimum of 10. Dayan and Zemel report that an AA trained without FMS (and more than 10 HUs) "consistently failed". This result has been confirmed by Baumgartner (1996).

For each of the 25 pixels there is an input unit. Input units that see a pixel of a bar take on activation 0.5, others -0.5. See Figure 1 for an example. Following Dayan and Zemel (1995), the net is trained on 500 randomly generated patterns (there may be pattern repetitions). Learning is stopped after 5,000 epochs. We say that a pattern is processed correctly if the absolute error of all output units is below 0.3.

Details. The sigmoid HUs are active in [0,1], the sigmoid output units are active in [-1,1]. Noninput units have an additional bias input. The target is -0.7 for -0.5 and 0.7 for 0.5 (to avoid tiny derivatives caused by targets -1.0 and 1.0). Normal weights are initialized in $[-0.1, 0.1]$, bias weights with -1.0, λ with 0.5. *Parameters:* learning rate: 1.0, $E_{tol} = 0.16$, $\Delta\lambda = 0.001$. *Architecture:* (25-25-25).

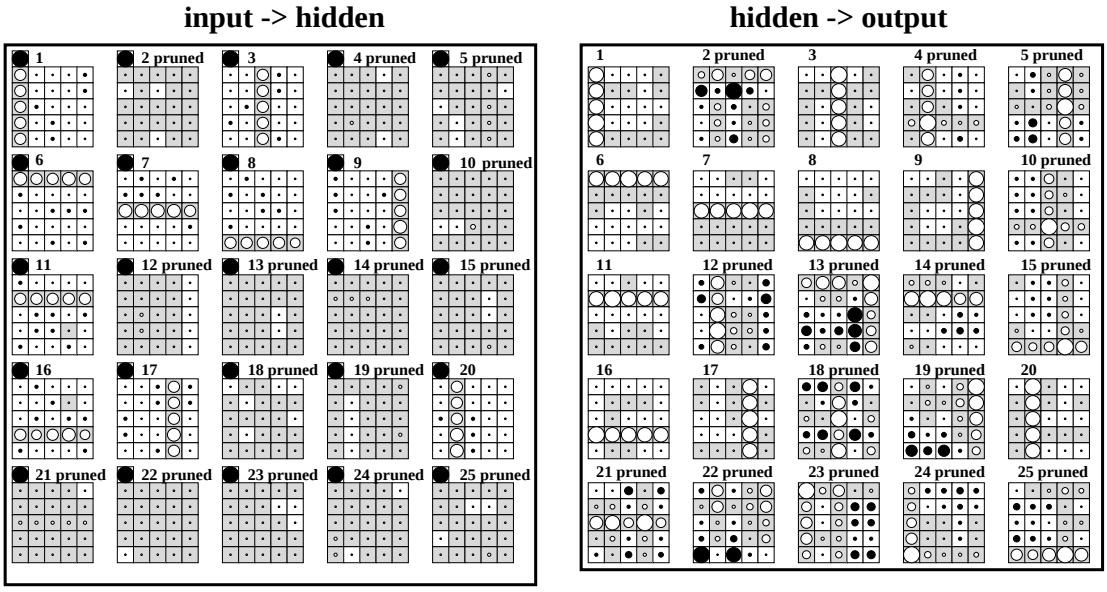


Figure 2: *Task 2.1 (independent bars). Left:* LOCOCODE*'s input-to-hidden weights. Right: hidden-to-output weights. See text for visualization details.*

Results: factorial (but sparse) codes. Training MSE is 0.11; the reconstruction error 0.081 (averages over 10 trials). The net generalizes well: only one of the test patterns is not processed correctly. 15 of the 25 HUs are indeed automatically pruned. All remaining HUs are binary: LOCOCODE finds an optimal factorial code which exactly mirrors the pattern generation process. Since the expected number of bars per input is 2, the code is also sparse.

For each of the 25 HUs, Figure 2 (left) shows a 5×5 square depicting 25 typical post-training weights on connections from 25 inputs (right: to 25 outputs). White (black) circles on gray (white) background are positive (negative) weights. The circle radius is proportional to the weight's absolute value. Figure 2 (left) also shows the bias weights (on top of the squares' upper left corners). The circle representing some HU's maximal absolute weight has maximal possible radius (circles representing other weights are scaled accordingly). Bias weights to HUs are in the in-

terval $[-8.5, -5.6]$; other weights to HUs are in $[-0.3, 4.4]$; bias weights to output units are in $[-1.85, -1.80]$, weights from hidden to output units are in $[-1.4, 3.8]$.

Backprop fails. For comparison we run this task with conventional BP with 25, 15 and 10 HUs. With 25 (15, 10) HUs the reconstruction error is 0.19 (0.24, 0.31). Backprop does not prune any units; the resulting weight patterns are highly unstructured, and the underlying input statistics are not discovered.

PCA and ICA. We tried both 10 and 15 components. Figure 3 shows results. PCA produces an unstructured and dense code, ICA-10 an almost sparse code where some sources are recognizable but not separated. ICA-15 finds a dense code and no sources. The reconstruction errors are: PCA-10: 0.086, PCA-15: 0.155, ICA-10: 0.084, ICA-15: 0.093. ICA/PCA codes with 10 components convey the same information as 10-component lococodes. The higher reconstruction errors for PCA-15 and ICA-15 are due to overfitting (the backprop net over-specializes on the training set).

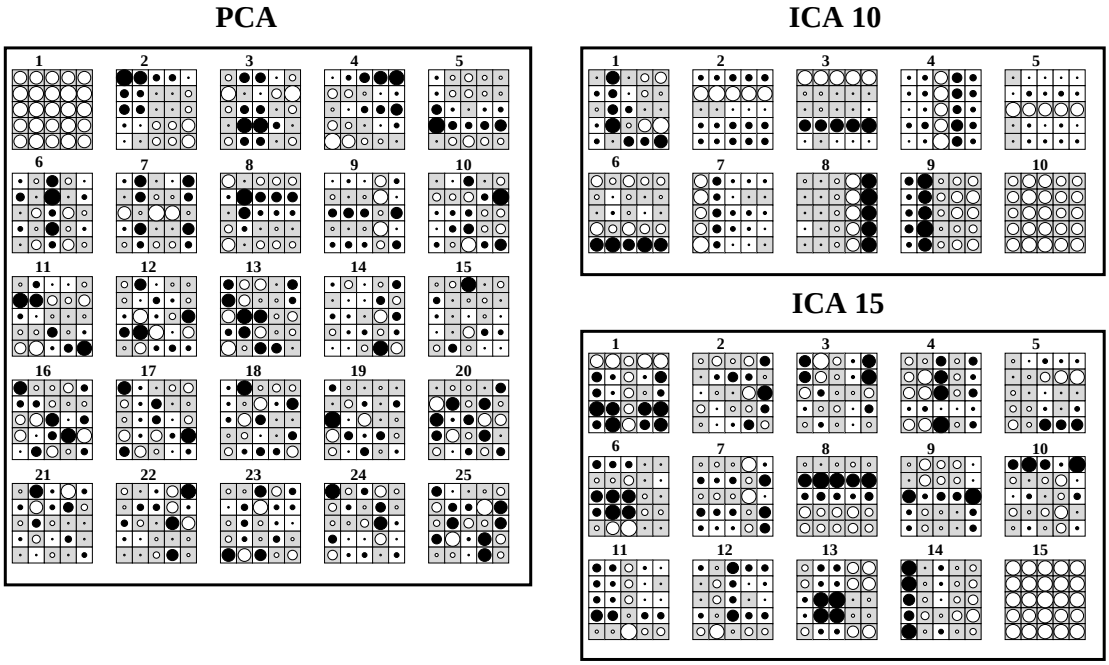


Figure 3: *Task 2.1 (independent bars). PCA and ICA: weights to code components (ICA with 10 and 15 components). ICA-10 does make some sources recognizable, but does not achieve lococode quality.*

Task 2.2 (noisy bars). Like Task 2.1 except for additional noise: bar intensities vary in $[0.1, 0.5]$; input units that see a pixel of a bar are activated correspondingly (recall the constant intensity 0.5 in Task 2.1), others adopt activation -0.5. We also add Gaussian noise with variance 0.05 and mean 0 to each pixel. Figure 4 shows some training exemplars generated in this way. The task is adapted from Hinton et al. (1995) and Hinton and Ghahramani (1997) but more difficult because vertical and horizontal bars may be mixed in the same input.

Details. Training, testing, coding and learning are as in Task 2.1, except that $E_{tol} = 2.5$ and $\Delta\lambda = 0.01$. E_{tol} is set to 2 times the expected minimal squared error: $E_{tol} = 2$*(number of inputs)*$\sigma^2 = 2 * 25 * 0.05 = 2.5$. To achieve consistency with Task 2.1, the target pixel value is 1.4 times the input pixel value (compare Task 2.1: $0.7 = 1.4 * 0.5$). All other learning parameters are like in Task 2.1.

Results. Training MSE is 2.5; the reconstruction error is 1.05 (averages over 10 trials), the net generalizes well. 15 of the 25 HUs are pruned away. Again LOCOCODE extracts an optimal (factorial) code which exactly mirrors the pattern generation process. Due to the bar intensity variations the remaining HUs are not binary as in Task 2.1.

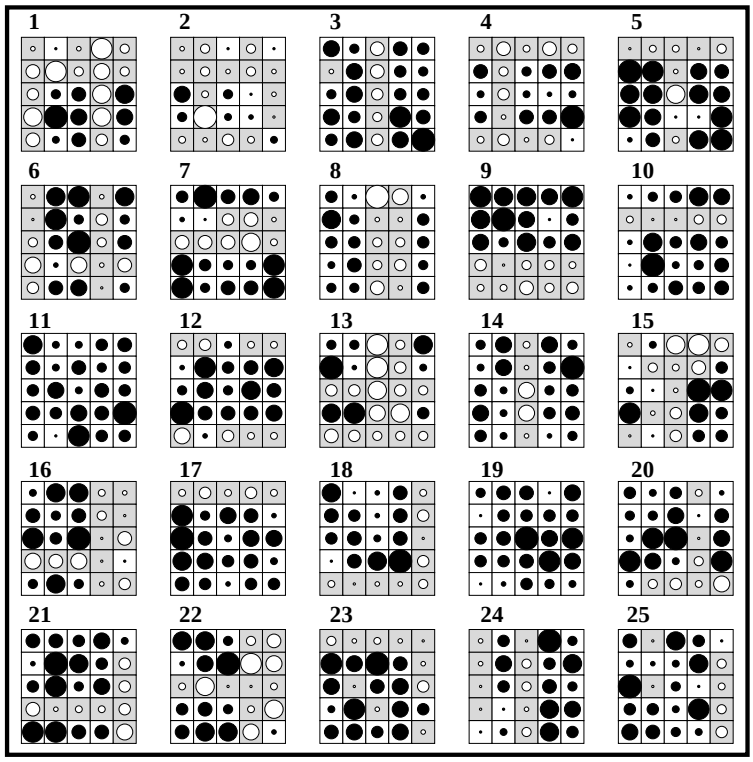


Figure 4: *Task 2.2 — noisy bars examples: 25* 5×5 *training inputs, depicted similarly to the weights in previous figures.*

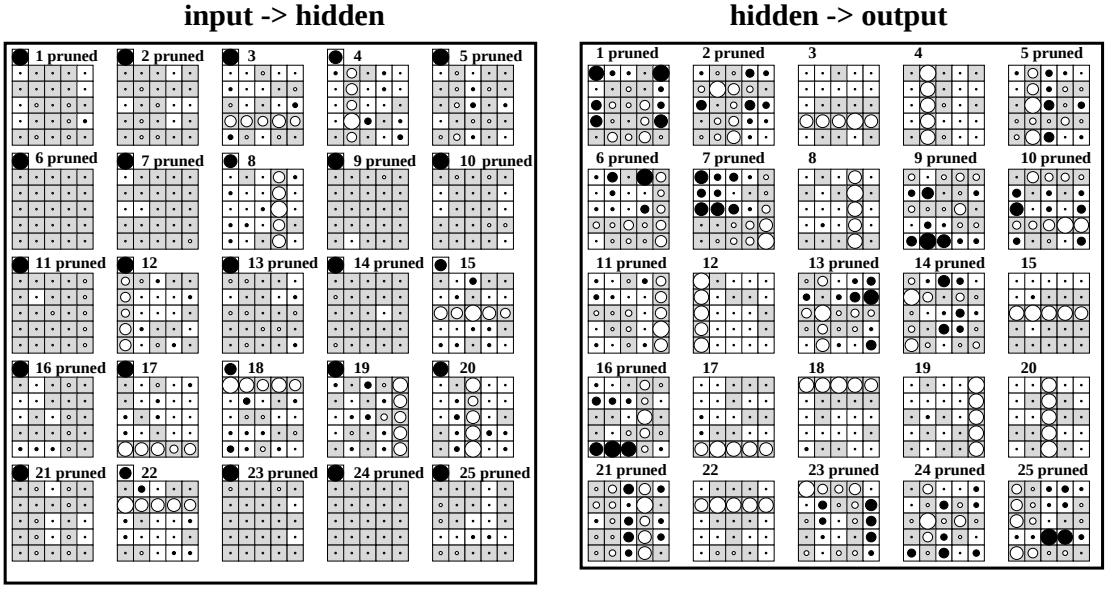


Figure 5: *Task 2.2 (independent noisy bars). Left:* LOCOCODE*'s input-to-hidden weights. Right: hidden-to-output weights.*

Figure 5 depicts typical weights to and from HUs. Bias weights to HUs are in $[-7.6, -1.9]$; other weights to HUs are in $[-1.8, 4.3]$; bias weights to the output units are in $[-2.1, -1.4]$; weights from HUs to output units are in $[-2.1, 5.3]$.

PCA and ICA. Figure 6 shows results comparable to those of Task 2.1. PCA codes and ICA-15 codes are unstructured and dense. ICA-10 codes, however, are almost sparse — some sources are recognizable. They are not separated though. The reconstruction errors are: PCA-10: 1.03, PCA-15: 0.72, ICA-10: 1.02, ICA-15: 0.71. We observe that PCA/ICA codes with 10 components convey as much information as 10-component lococodes. The lower reconstruction error for PCA-15 and ICA-15 is due to information about the current noise conveyed by the additional code components (we reconstruct noisy images).

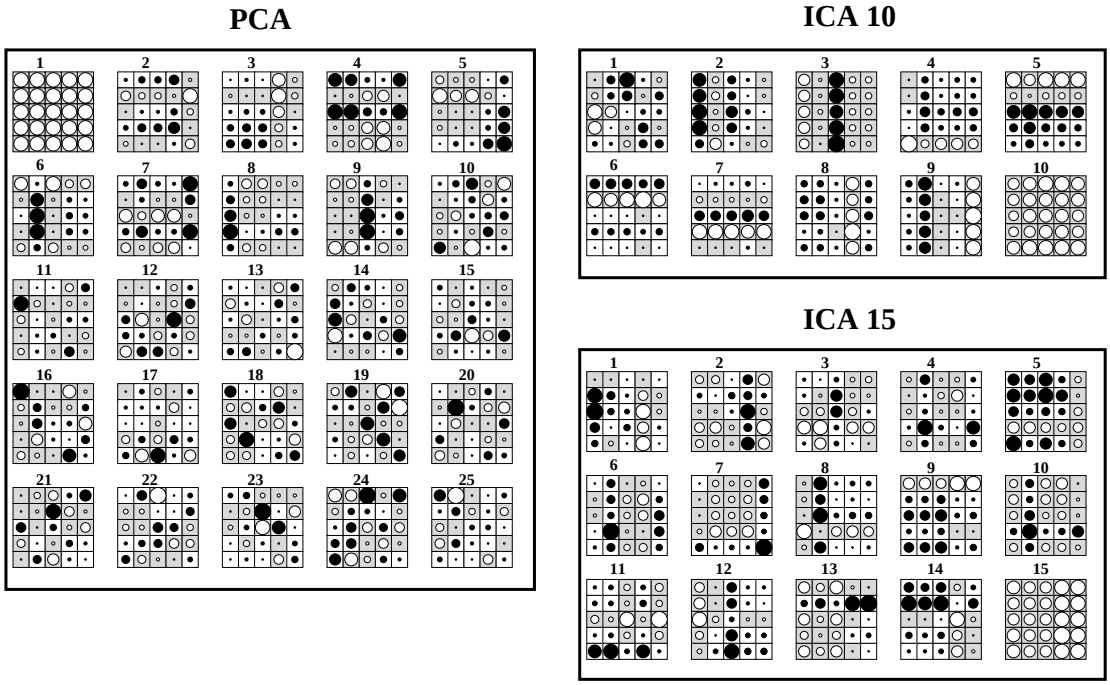


Figure 6: *Task 2.2 (independent noisy bars). PCA and ICA: weights to code components (ICA with 10 and 15 components). Only ICA-10 codes extract a few sources, but they do not achieve the quality of lococodes.*

Conclusion. LOCOCODE solves a hard variant of the standard "bars" problem. It discovers the underlying statistics and extracts the essential, statistically independent features, even in presence of noise. Standard BP AAs accomplish none of these feats (Dayan and Zemel, 1995) — this has been confirmed by additional experiments conducted by ourselves. ICA and PCA also fail to extract the true input causes and the optimal features.

LOCOCODE achieves success solely by reducing information-theoretic (de)coding costs. Unlike previous approaches, it does not depend on explicit terms enforcing independence (e.g., Schmidhuber 1992), zero mutual information among code components (e.g., Linsker 1988, Deco and Parra 1994), or sparseness (e.g., Field 1994, Zemel and Hinton 1994, Olshausen and Field 1996, Zemel 1993, Hinton and Ghahramani 1997).

LOCOCODE vs. ICA. Like recent simple methods for "independent component analysis" (ICA, e.g., Cardoso and Souloumiac 1993, Bell and Sejnowski 1995, Amari et al. 1996) LOCOCODE untangles mixtures of independent data sources. Unlike these methods, however, it does not need to know in advance the number of such sources — like "predictability minimization" (a nonlinear ICA approach — Schmidhuber 1992), it simply prunes away superfluous code components.

In many visual coding applications few sources determine the value of a given output (input) component, and the sources are easily computable from the input. Here LOCOCODE outperforms simple ICA because it minimizes the number of low-complexity sources responsible for each output component.

4.4 EXPERIMENT 3: More Realistic Visual Data

Task 3.1a. As in Experiment 2 the goal is to extract features from visual data. The input data is more realistic though — we use the aerial shot of a village.

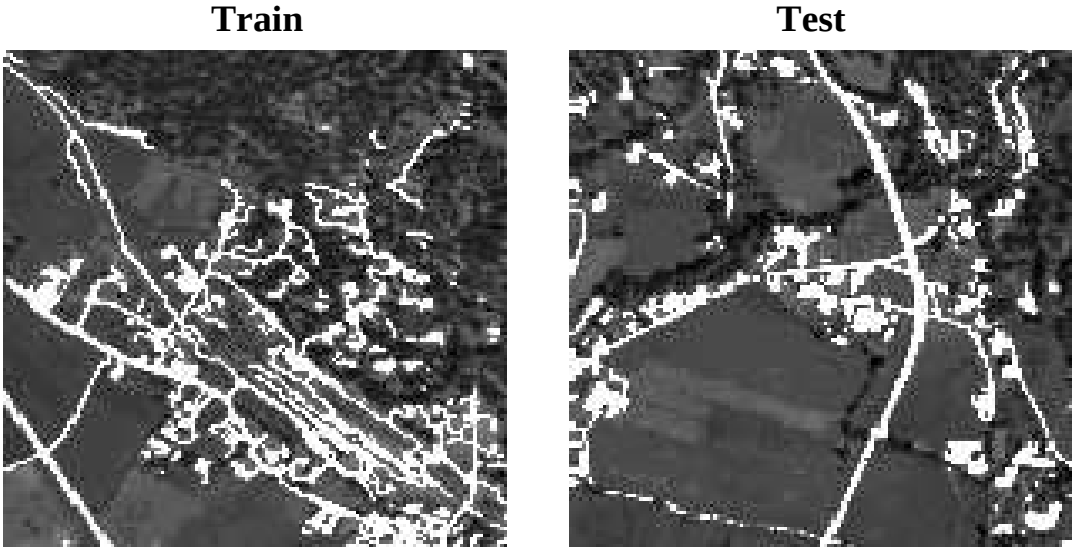


Figure 7: *Task 3.1 — village image. Image sections used for training (left) and testing (right).*

Details. Figure 7 shows two images with 150×150 pixels, each taking on one of 256 gray levels. 5×5 pixels subsections from the left hand side (right hand side) image are randomly chosen as training inputs (test inputs), where gray levels are scaled to input activations in $[-0.5, 0.5]$. The sigmoid HUs are active in [0,1], the sigmoid output units in [-1,1]. Targets are scaled to $[-0.7, 0.7]$. Normal weights are initialized in $[-0.1, 0.1]$, bias weights (to noninput units) with -1.0, λ with 0.5. Training stop: after 300,000 training examples. *Parameters:* learning rate: 1.0, $E_{tol} = 1.0$, $\Delta\lambda = 0.05$. *Architecture:* (25-25-25).

Image structure. The image is mostly dark except for certain white regions. In a preprocessing stage we map pixel values above 119 to 255 (white) and pixel values below 120 to 9 different gray values. The largest reconstruction errors will be due to absent information about white pixels. Our receptive fields are too small to capture structures such as lines (streets).

Result: sparse codes, on-center-off-surrounds. 6 trials led to similar results (6 trials seem sufficient due to tiny variance). Only 8 to 10 out of 25 HUs survive. They indeed reflect the structure of the image (compare the preprocessing stage): (1) Informative white spots are captured by on-center-off-surround HUs. (2) Since the image is mostly dark (this also causes the off-surround effect), all output units are negatively biased. (3) Since most bright spots are connected (most white pixels are surrounded by white pixels), output/input units near an active output/input unit tend to be active, too (positive weight strength decreases as one moves away from the center). (4) The entire input is covered by on-centers of surviving units — all white regions in the input will be detected. (5) The code is sparse: few surviving white-spot-detectors are active simultaneously because most inputs are mostly dark. The reconstruction error is 1.05.

Figure 8 depict typical weights on connections to and from HUs. Bias weights to HUs and output units are negative. To activate some HU, its input must match the structure of the incoming weights. Most HUs are pruned, 8 survive.

Backprop results. Compare all of this to nets taught by standard BP — see Figure 9 for a typical example. The reconstruction errors for 25 (15, 10) HU are 0.88 (1.12, 1.31). There are hardly any recognizable structures, and none of the HUs is close to being pruned away.

PCA and ICA. Figure 10 shows results for PCA and ICA. The reconstruction errors are: PCA-8: 1.04, PCA-10: 0.97, ICA-8: 1.04, ICA-10: 1.11. Codes obtained by PCA-8 and ICA-

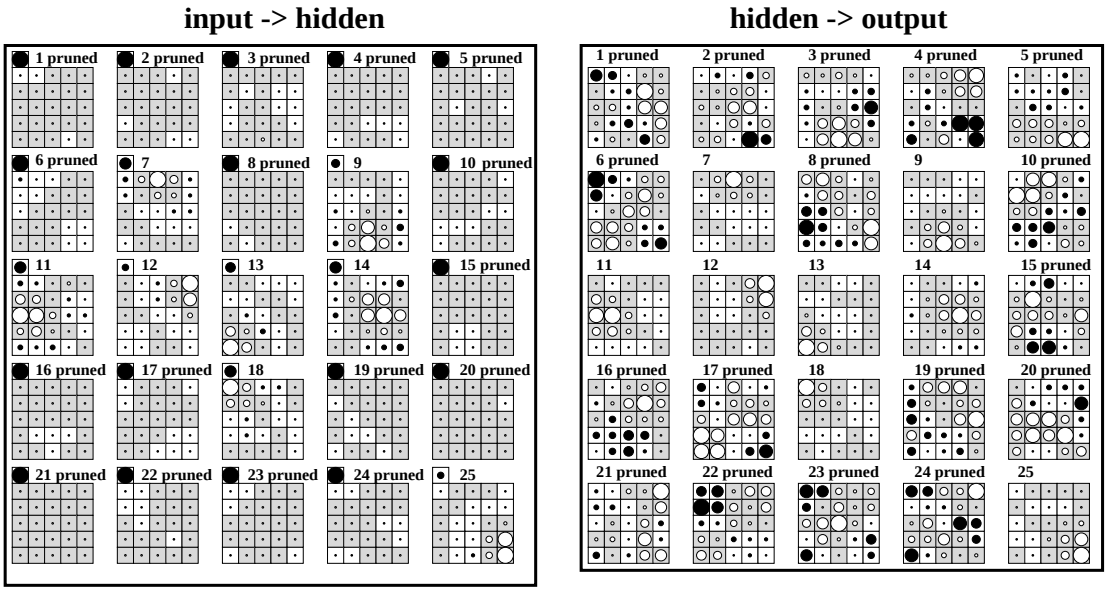


Figure 8: *Task 3.1a (village). Left:* LOCOCODE*'s input-to-hidden weights. Right: hidden-to-output weights. Most units are essentially pruned away.*

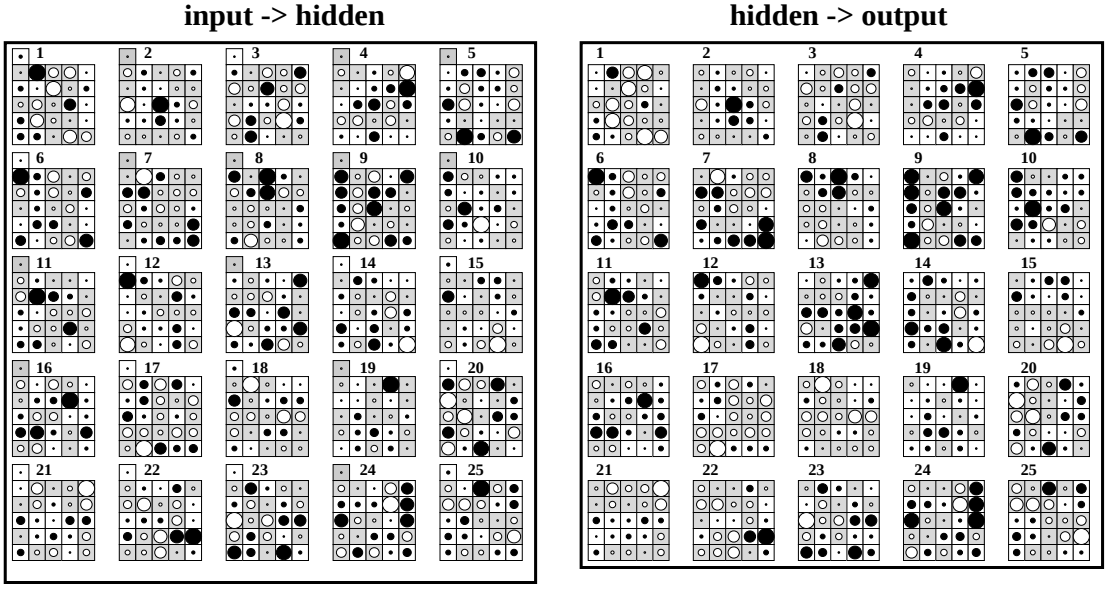


Figure 9: *Task 3.1a (village). Left: standard backprop's input-to-hidden weights. Right: hidden-to-output weights. There are no pruned units and hardly any recognizable structures.*

8 convey as much information as lococodes with 8 components. The eigenvalues of the PCA components are: 0.7994, 0.2222, 0.1497, 0.1268, 0.0759, 0.0697, 0.0613, 0.0496, 0.0389, 0.0333, 0.0291, 0.0286, 0.0245, 0.0229, 0.0197, 0.0187, 0.0168, 0.0161, 0.0156, 0.0145, 0.0134, 0.0129, 0.0112, 0.0102, 0.0082. The last significant difference between subsequent eigenvalues (> 0.01) occurs between the 8th and the 9th. This indicates that there should be 8 code components. LOCOCODE discovers this automatically. The lococode codes white spots and seems to suit the image better than PCA's or ICA's code component structures.

Task 3.1b. Like Task 3.1a, except: 7×7 pixels subsections are randomly chosen as training and

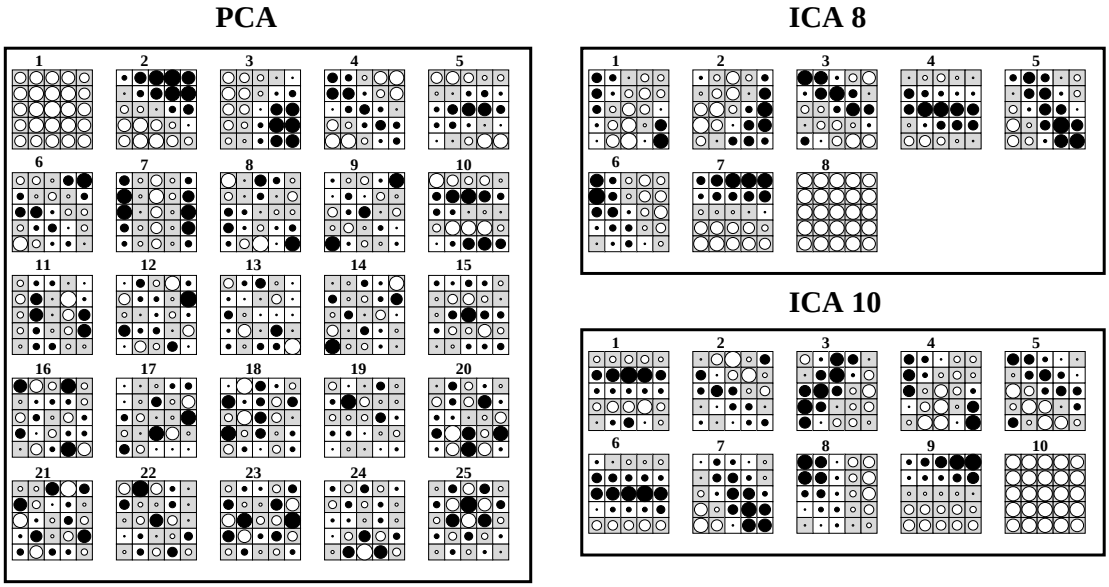


Figure 10: *Task 3.1a (village). PCA and ICA: weights to code components (ICA with 8 and 10 components).*

test inputs, $E_{tol} = 3.0$, architecture: (49-25-49). Training stop: after 150,000 training examples. All other parameters are like in Task 3.1a.

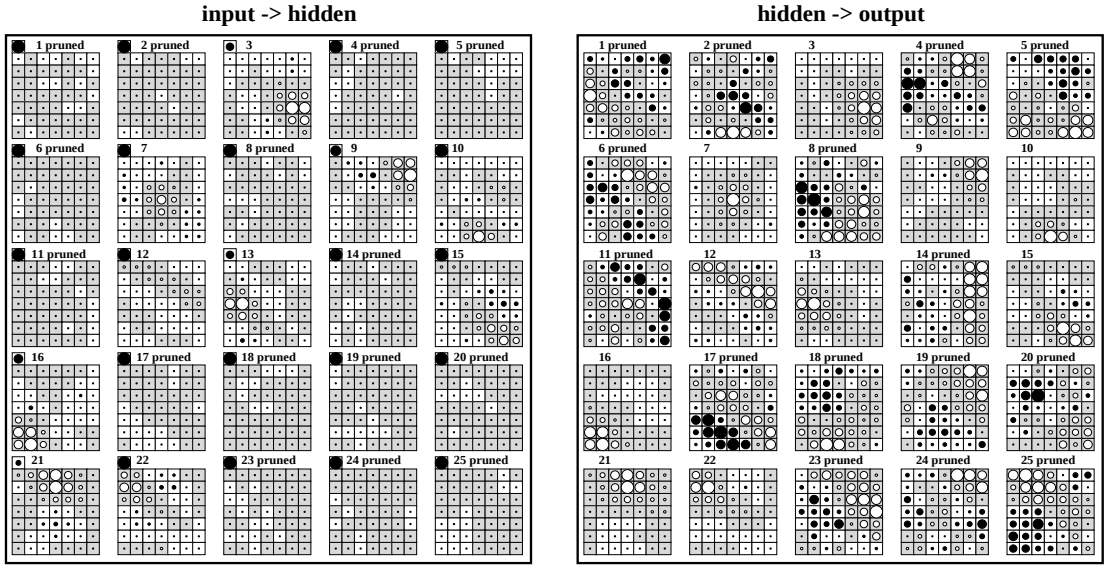


Figure 11: *Task 3.1b (village). Left:* LOCOCODE*'s input-to-hidden weights. Right: hidden-to-output weights. Most units are essentially pruned away.*

Results: sparse codes, on-center-off-surrounds. 6 trials led to results similar to Task 3.1a's (6 trials seem sufficient due to tiny variance). Only 9 to 11 HUs survive. Again, the entire input is covered by white on-centers of surviving units that exhibit on-center-off-surround weight structures. This allows for detecting all white regions in the input field. Again, the code is sparse (compare Task 3.1a). The reconstruction error is 8.29. Figure 11 depicts typical weights on connections to and from HUs (output units are negatively biased). 10 units survive.

Task 3.1b leads to larger on-centers than Task 3.1a, which typically provokes on-centers corresponding to one or two weights.

PCA and ICA. Figure 12 shows results for PCA and ICA. Reconstruction errors are: PCA-10: 9.21, PCA-15: 8.03, ICA-10: 7.90, ICA-15: 6.57. PCA-10 codes and ICA-10 codes are about as informative as 10 component lococodes (ICA-10 a bit more and PCA-10 less). PCA-15 codes convey no more information: LOCOCODE and ICA suit the image structure better. PCA's eigenvalues are: 0.0907, 0.0725, 0.0263, 0.0095, 0.0039, 0.0019, 0.0014, 0.0010, 0.0009, 0.0008, 0.0008, 0.0006, 0.0006, 0.0006, 0.0005, 0.0005, 0.0005, 0.0004, 0.0004, 0.0003, 0.0002, 0.0002, 0.0002, 0.0002, 0.0002, 0.0002, 0.0002, 0.0002, 0.0002, 0.0002. There is no significant difference between subsequent eigenvalues after the 8th.

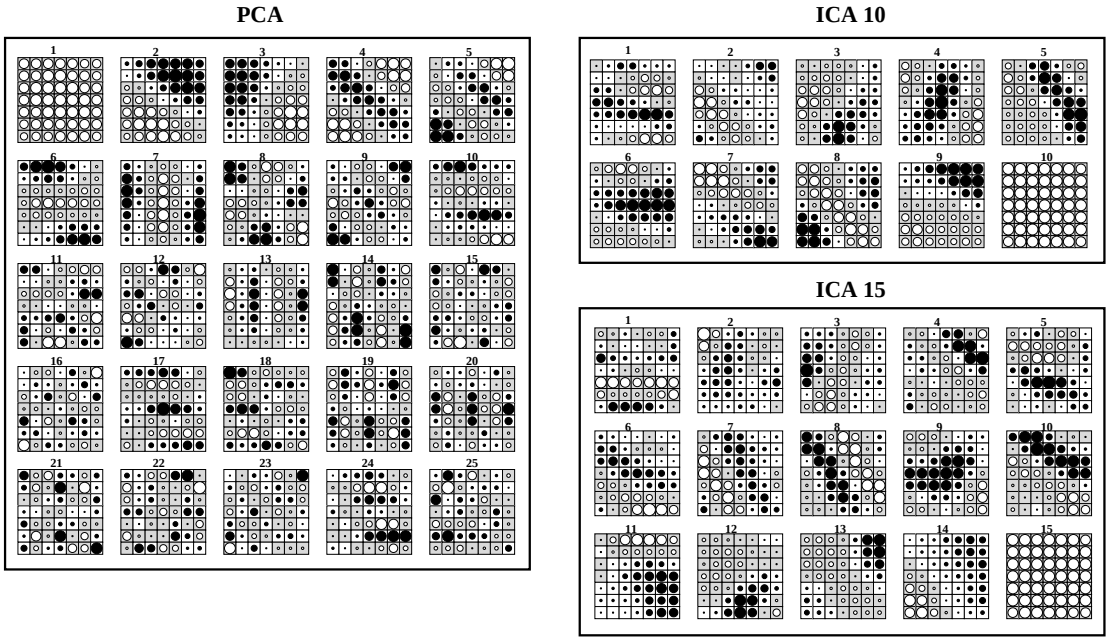


Figure 12: *Task 3.1b (village). PCA and ICA (with 10 and 15 components): weights to code components.*

Task 3.2. Like Task 3.1, but the inputs stem from a 150×150 pixels section of an image of wood cells (Figure 13: left: training image, right: test image). $E_{tol} = 1.0$, $\Delta\lambda = 0.01$. Training stop: after 250,000 training examples. All other parameters are like in Task 3.1.

Image structure. The image consists of elliptic cells of various sizes. Cell interiors are bright; cell borders dark.

Results. 4 trials led to similar results (4 trials seem sufficient due to tiny variance). Bias weights to HUs are negative. To activate some HU, its input must match the structure of the incoming weights to cancel the inhibitory bias. 9 to 11 units survive. They are obvious feature detectors and can be characterized by the positions of the centers of their on-center-off-surround structures relative to the input field. They are specialized on detecting the following cases: the on-center is north, south, west, east, northeast, northwest, southeast, southwest of a cell, or centered on a cell, or between cells. Hence the entire input is covered by position-specialized on-centers. The reconstruction error is 0.840.

Figure 14 depicts typical weights on connections to and from HUs after 50,000 training examples. Figure 15 shows that further learning (200,000 additional examples) makes the system concentrate on on-centers (left hand side of Figure 15); negative weights become less pronounced than positive ones (right hand side of Figure 15). Longer training (1) makes the code sparser and

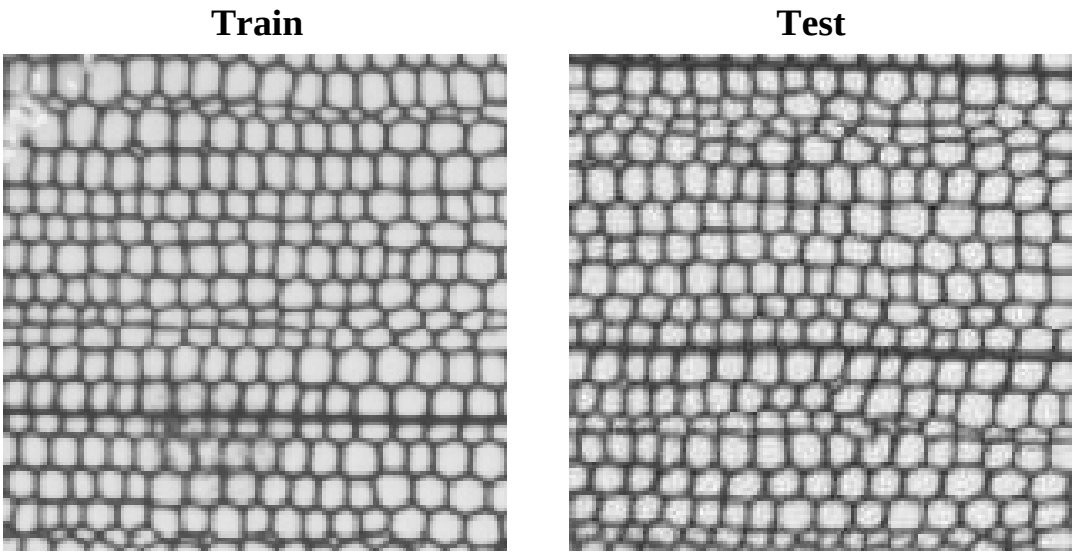


Figure 13: *Task 3.2 — wood cells. Image sections used for training (left) and testing (right).*

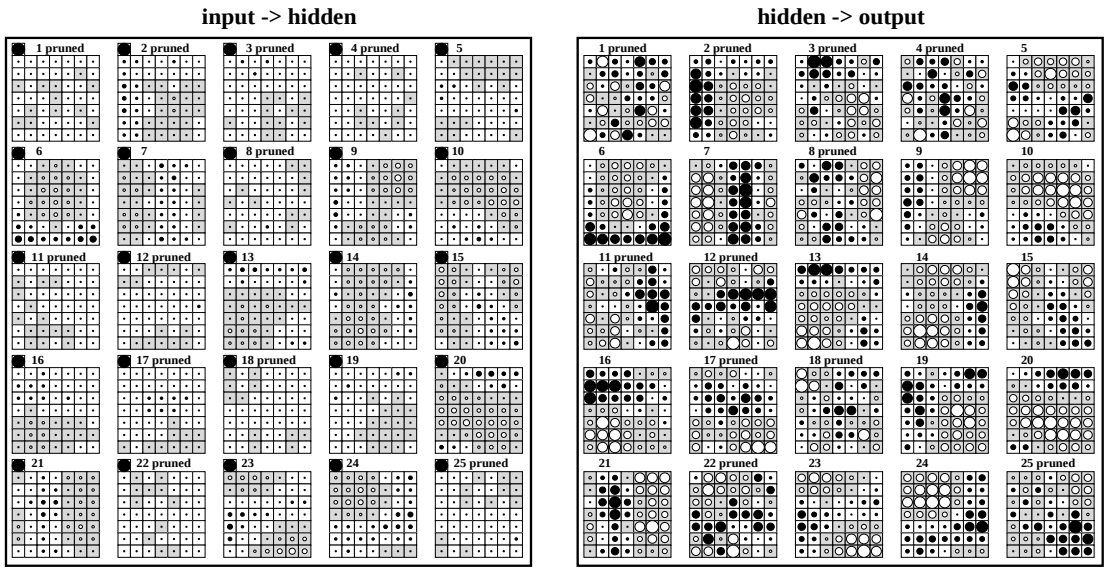


Figure 14: *Task 3.2 (cells). Left:* Lococode*'s input-to-hidden weights after 50,000 exemplars. Right: hidden-to-output weights. 14 units survive.*

(2) causes more pruned units. Typical feature detectors: unit 20 detects a southeastern cell; unit 21 western and eastern cells; unit 23 cells in the northwest and southeast corners.

PCA and ICA. Figure 16 shows results for PCA and ICA. Reconstruction errors are: PCA-11: 0.722, PCA-15: 0.329, ICA-11: 0.871, ICA-15: 0.360. PCA-11 codes and ICA-11 are about as informative as the 11 component lococode (ICA-11 a little less and PCA-11 more). Component structures of ICA codes and lococodes are very similar: compare component 24 in Figure 15 and component 4 for ICA-11 (component 8 for ICA-15) in Figure 16; compare component 14 in Figure 15 and component 3 for ICA-11 (component 5 for ICA-15) in Figure 16; compare component 23 in Figure 15 and component 6 for ICA-11 (component 4 for ICA-15) in Figure 16. It seems that both Lococode and ICA detect relevant sources: the positions of the cell interiors (and cell borders) relative to the input field. The eigenvalues of the PCA components are: 0.3584, 0.3236, 0.3083,

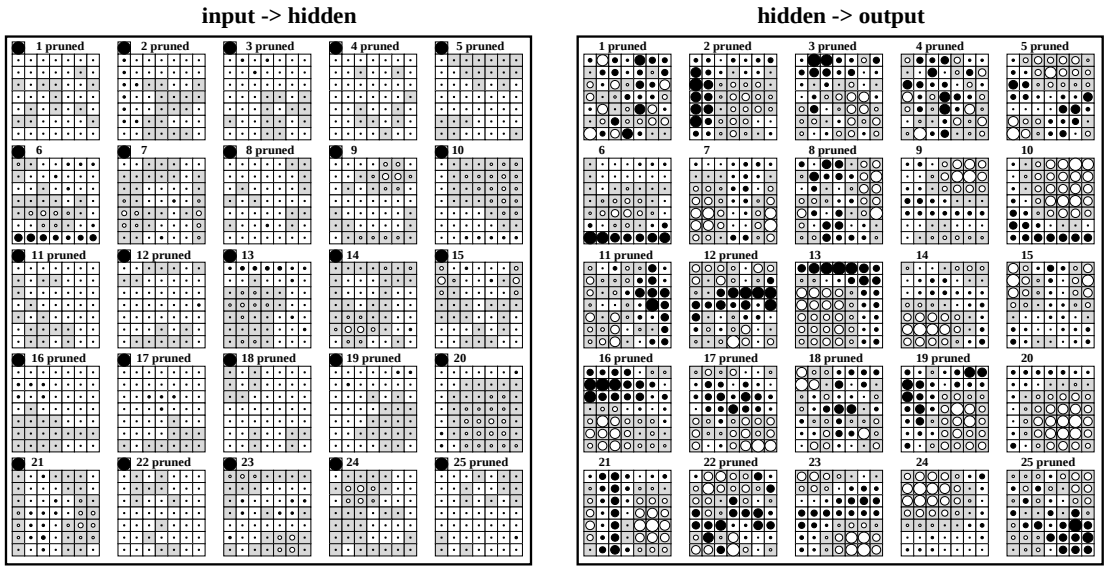


Figure 15: *Task 3.2 (cells). Left:* Lococode*'s input-to-hidden weights after 250,000 exemplars. Right: hidden-to-output weights. 11 units survive.*

0.1764, 0.1324, 0.1151, 0.0716, 0.0578, 0.0494, 0.0451, 0.0254, 0.0221, 0.0218, 0.0191, 0.0181, 0.0103, 0.0088, 0.0084, 0.0081, 0.0068, 0.0067, 0.0066, 0.0057, 0.0040, 0.0034, 0.0031, 0.0030, 0.0027, 0.0024, 0.0023, 0.0017, 0.0013, 0.0012, 0.0011, 0.0010, 0.0008, 0.0006, 0.0005, 0.0004, 0.0004, 0.0004, 0.0003, 0.0002, 0.0002, 0.0002, 0.0001, 0.0001, 0.0001, 0.0001. Gaps occur between the 10th and the 11th, and between the 15th and the 16th. Lococode essentially found the first gap.

Task 3.3. Like Task 3.1 — but now we use images of *striped* piece of wood. See Figure 17. $E_{tol} = 0.1$. Training stop: after 300,000 training examples. All other parameters are like in Task 3.1.

Image structure. The image consists of dark vertical stripes on a brighter background.

Results. 4 trials led to similar results Only 3 to 5 of the 25 HUS survive and become obvious feature detectors, now of a different kind: they detect whether their receptive field covers a dark stripe to the left, to the right, or in the middle. The reconstruction error is 0.831.

Figure 18 (after 100,000 examples) and Figure 19 (after 300,000 examples) depict typical weights on connections to and from HUs. Example feature detectors: unit 6 detects a dark stripe to the left, unit 11 a dark stripe in the middle, unit 15 dark stripes left and right, unit 25 a dark stripe to the right. Unit 2 in Figure 18 becomes less and less informative until it finally gets pruned away (Figure 19).

PCA and ICA. See Figure 20. Reconstruction errors are: PCA-4: 0.830, PCA-10: 0.534, ICA-4: 0.856, ICA-10: 0.716. PCA-4 codes and ICA-4 codes are about as informative as 4-component lococodes. Component structures of PCA/ICA codes and lococodes are very similar: all detect the positions of dark stripes relative to the input field. The eigenvalues of the PCA components are: 0.0907, 0.0725, 0.0263, 0.0095, 0.0039, 0.0019, 0.0014, 0.0010, 0.0009, 0.0008, 0.0008, 0.0006, 0.0006, 0.0006, 0.0005, 0.0005, 0.0005, 0.0004, 0.0004, 0.0003, 0.0002, 0.0002, 0.0002, 0.0002, 0.0002, 0.0002, 0.0002, 0.0002, 0.0002, 0.0002. Gaps occur between 3rd and 4th, 4th and 5th, 5th and 6th. Lococode automatically extracts about 4 relevant components.

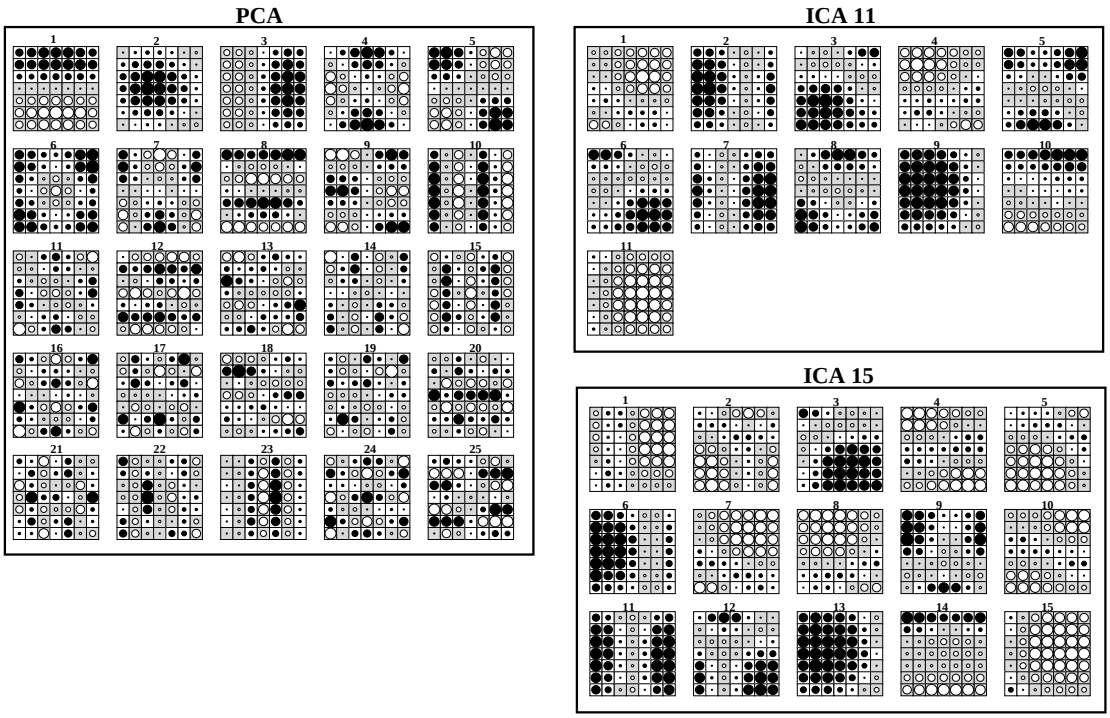


Figure 16: *Task 3.2 (cells). PCA and ICA (with 11 and 15 components): weights to code components.*

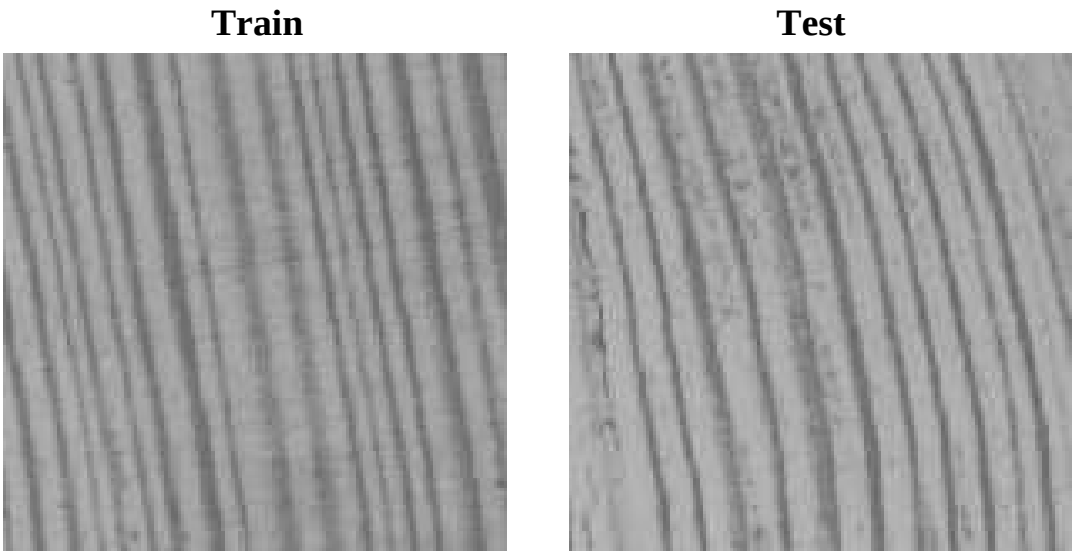


Figure 17: *Task 3.3 — striped wood. Image sections used for training (left) and testing (right).*

4.4.1 Overview over experiments 2 and 3

Table 2 shows that most lococodes and some ICA codes are sparse, while most PCA codes are dense. Assuming that each visual input consists of many components collectively describable by few input features, LOCOCODE seems preferrable.

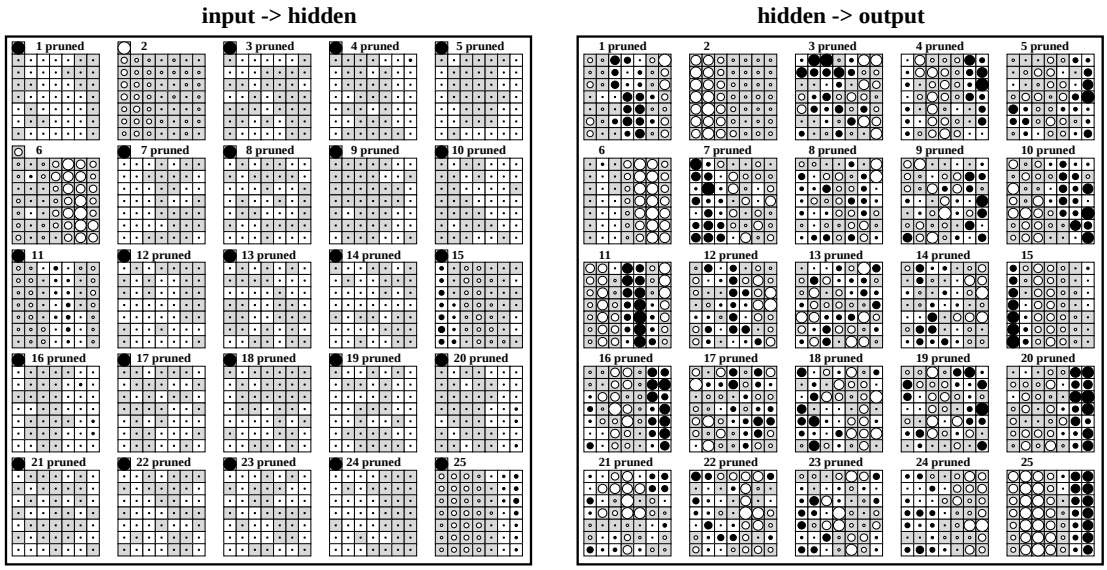


Figure 18: *Task 3.3 (stripes). Left:* Lococode*'s input-to-hidden weights after 100,000 exemplars. Right: hidden-to-output weights. 5 units survive.*

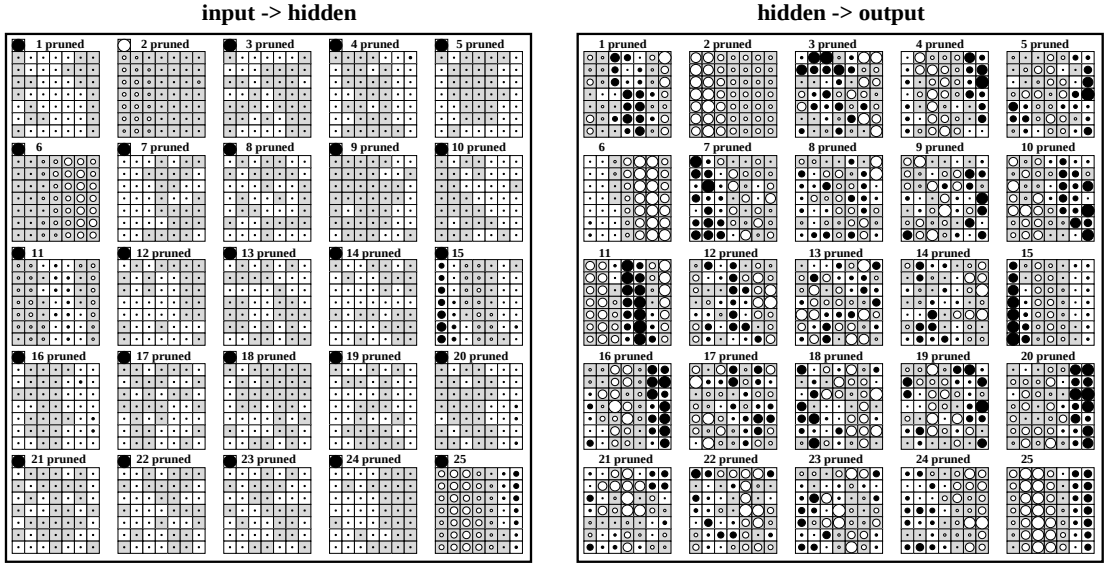


Figure 19: *Task 3.3 (stripes). Left:* Lococode*'s input-to-hidden weights after 300,000 exemplars. Right: hidden-to-output weights. 4 units survive.*

Conclusion. Unlike standard BP-trained AAs, FMS-trained AAs generate highly structured sensory codes. FMS automatically prunes superfluous units. PCA experiments indicate that the remaining code units suit the various coding tasks well. Taking into account statistical properties of the visual input data, Lococode generates appropriate feature detectors such as the familiar on-center-off-surround and bar detectors. It also produces biologically plausible sparse codes (standard AAs do not). FMS's objective function, however, does *not* contain explicit terms enforcing such codes (this contrasts previous methods, e.g., Olshausen and Field 1996).

The experiments show that equally-sized PCA codes, ICA codes, and lococodes convey approximately the same information. In contrast to PCA and ICA, however, Lococode determines

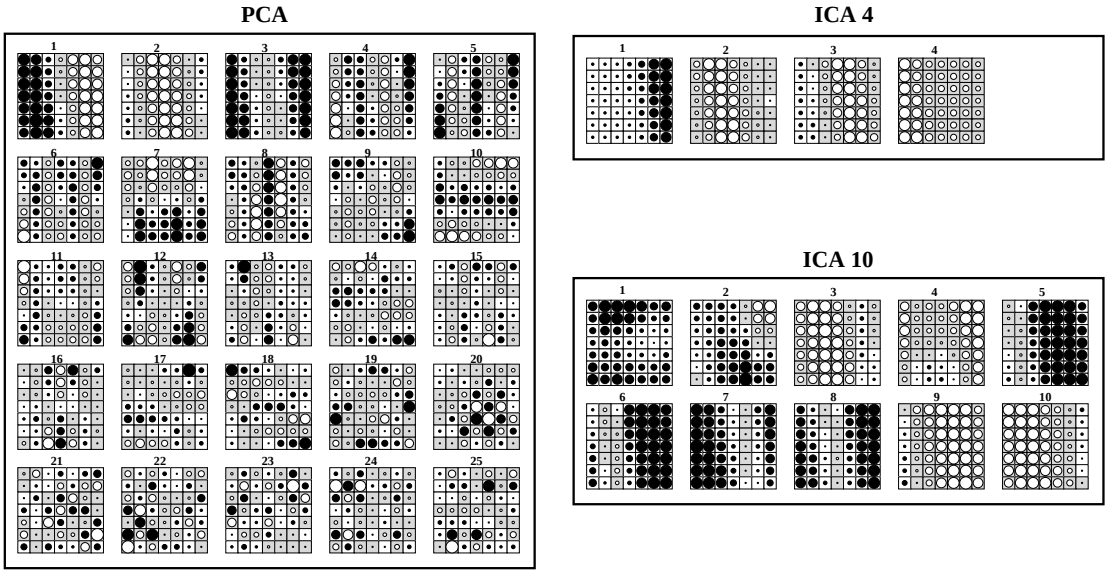


Figure 20: *Task 3.3 (stripes). PCA and ICA (with 11 and 15 components).*

the code size automatically. Some of the feature detectors obtained by LOCOCODE are similar to those found by ICA. In cases where we *know* the true input causes, however, LOCOCODE does find them whereas ICA does not.

4.5 EXPERIMENT 4: vowel recognition

Lococodes cannot only be justified by reference to previous ideas on what's a "desirable" code. Next we will show that they can help to achieve superior generalization performance on a standard supervised learning benchmark problem. This section's focus on speech data also illustrates LOCOCODES's versatility: its applicability is not limited to visual data.

Task. We recognize vowels, using vowel data from Scott Fahlman's CMU benchmark collection (see also Robinson 1989). There are 11 vowels and 15 speakers. Each speaker spoke each vowel 6 times. Data from the first 8 speakers is used for training. The other data is used for testing. This means 528 frames for training and 462 frames for testing. Each frame consists of 10 input components obtained by low pass filtering at 4.7kHz, digitized to 12 bits with a 10 kHz sampling rate. A twelfth order linear predictive analysis was carried out on six 512 sample Hamming-windowed segments from the steady part of the vowel. The reflection coefficients were used to calculate 10 log area parameters, providing the 10 dimensional input space.

Coding. The training data is coded using an FMS AA. Architecture: (10-30-10). The sigmoid HUs are active in [0,1], the sigmoid output units are active in [-1,1]. Noninput units have an additional bias input. The input components are linearly scaled in [-1,1]. The AA is trained with 10^7 pattern presentations. Then its weights are frozen.

Classification. From now on, the vowel codes across all nonconstant HUs are used as inputs for a conventional supervised BP classifier, which is trained to recognize the vowels from the code. The classifier's architecture is $((30-c)$-11-11), where c is the number of nonvarying (pruned) HUs in the AA. The hidden and output units are sigmoid and active in [-1,1], and receive an additional bias input. The classifier is trained with another 10^7 pattern presentations.

Parameters. AA net: learning rate: 0.02, $E_{tol} = 0.015$, $\Delta\lambda = 0.2$, $\gamma = 2.0$. Backprop classifier: learning rate: 0.002.

Overfitting. We confirm Robinson's results: the classifier tends to overfit when trained by simple BP — during learning, the test error rate first decreases and then increases again.

Exp.	input field	method	# code comp.	reconst. error	code type
bars	5×5	LOC	10	0.081	sparse (factorial)
bars	5×5	ICA	10	0.084	almost sparse
bars	5×5	PCA	10	0.086	dense
bars	5×5	ICA	15	0.093	dense
bars	5×5	PCA	15	0.155	dense
noisy bars	5×5	LOC	10	1.05	sparse (factorial)
noisy bars	5×5	ICA	10	1.02	almost sparse
noisy bars	5×5	PCA	10	1.03	dense
noisy bars	5×5	ICA	15	0.71	dense
noisy bars	5×5	PCA	15	0.72	dense
village image	5×5	LOC	8	1.05	sparse
village image	5×5	ICA	8	1.04	almost sparse
village image	5×5	PCA	8	1.04	dense
village image	5×5	ICA	10	1.11	almost sparse
village image	5×5	PCA	10	0.97	dense
village image	7×7	LOC	10	8.29	sparse
village image	7×7	ICA	10	7.90	dense
village image	7×7	PCA	10	9.21	dense
village image	7×7	ICA	15	6.57	dense
village image	7×7	PCA	15	8.03	dense
wood cell image	7×7	LOC	11	0.840	sparse
wood cell image	7×7	ICA	11	0.871	sparse
wood cell image	7×7	PCA	11	0.722	almost sparse
wood cell image	7×7	ICA	15	0.360	sparse
wood cell image	7×7	PCA	15	0.329	dense
wood piece image	7×7	LOC	4	0.831	almost sparse
wood piece image	7×7	ICA	4	0.856	almost sparse
wood piece image	7×7	PCA	4	0.830	almost sparse
wood piece image	7×7	ICA	10	0.716	almost sparse
wood piece image	7×7	PCA	10	0.534	almost sparse

Table 2: *Overview over experiments 2 and 3: name of experiment, input field size, coding method, number of relevant code components (code size), reconstruction error, nature of code observed on the test set. PCA's and ICA's code sizes are prewired.* LOCOCODE*'s, however, are found automatically.*

Comparison. We compare: *(1) Various neural nets* (see Table 1). *(2) Nearest neighbor:* classifies an item as belonging to the class of the closest example in the training set (using Euclidean distance). *(3) LDA:* linear discriminant analysis. *(4) Softmax:* observation assigned to class with best fit value. *(5) QAD:* quadratic discriminant analysis (observations are classified as belonging to the class with closest centroid, using Mahalanobis distance based on the class-specific covariance matrix). *(6) CART:* classification and regression tree (coordinate splits and default input parameter values are used). *(7) FDA/BRUTO:* flexible discriminant analysis using additive models with adaptive selection of terms and splines smoothing parameters. BRUTO provides a set of basis functions for better class separation. *(8) Softmax/BRUTO:* best fit value for classification using BRUTO. *(9) FDA/MARS:* FDA using multivariate adaptive regression splines. MARS builds a basis expansion for better class separation. *(10) Softmax/MARS:* best fit value for classification using MARS. *(11)* LOCOCODE*/Backprop:* "unsupervised" codes generated by LOCOCODE with FMS, fed into a conventional, overfitting BP classifier.

	Technique	nr. hidden units	error rates	
			training	test
(1.1)	Single-layer perceptron	–	–	0.67
(1.2.1)	Multi-layer perceptron	88	–	0.49
(1.2.2)	Multi-layer perceptron	22	–	0.55
(1.2.3)	Multi-layer perceptron	11	–	0.56
(1.3.1)	Modified Kanerva Model	528	–	0.50
(1.3.2)	Modified Kanerva Model	88	–	0.57
(1.4.1)	Radial Basis Function	528	–	0.47
(1.4.2)	Radial Basis Function	88	–	0.52
(1.5.1)	Gaussian node network	528	–	0.45
(1.5.2)	Gaussian node network	88	–	0.47
(1.5.3)	Gaussian node network	22	–	0.46
(1.5.4)	Gaussian node network	11	–	0.53
(1.6.1)	Square node network	88	–	0.45
(1.6.2)	Square node network	22	–	0.49
(1.6.3)	Square node network	11	–	0.50
(2)	Nearest neighbor	–	–	0.44
(3)	LDA	–	0.32	0.56
(4)	Softmax	–	0.48	0.67
(5)	QDA	–	0.01	0.53
(6.1)	CART	–	0.05	0.56
(6.2)	CART (linear comb. splits)	–	0.05	0.54
(7)	FDA / BRUTO	–	0.06	0.44
(8)	Softmax / BRUTO	–	0.11	0.50
(9.1)	FDA / MARS (degree 1)	–	0.09	0.45
(9.2)	FDA / MARS (degree 2)	–	0.02	0.42
(10.1)	Softmax / MARS (degree 1)	–	0.14	0.48
(10.2)	Softmax / MARS (degree 2)	–	0.10	0.50
(11)	LOCOCODE / Backprop	30/11	0.05	0.42

Table 3: *Vowel recognition task: generalization performance of different methods. Surprisingly, FMS-generated lococodes fed into a conventional, overfitting backprop classifier led to excellent results. See text for details.*

Results. See Table 3. FMS generates 3 different lococodes. Each is fed into 10 BP classifiers with different weight initializations: the table entry for "LOCOCODE/Backprop" represents the mean of 30 trials. The results for neural nets and nearest neighbor are taken from Robinson (1989). The other results (except for LOCOCODE's) are taken from Hastie et al. (1993). Our method led to excellent generalization results. The error rates after BP learning vary between 39 and 45 %.

Backprop fed with LOCOCODE code sometimes goes down to 38 % error rate, but due to overfitting, the error rate increases again (of course, test set performance may not influence the training procedure). Given that BP by itself is a very naive approach it seems quite surprising that excellent generalization performance can be obtained just by feeding BP with *nongoal-specific* lococodes.

Typical feature detectors. The number of pruned HUs (with constant activation) varies between 5 and 10. 2 to 5 HUs become binary, and 4 to 7 trinary. With all codes we observed: apparently, certain HUs become feature detectors for speaker identification. Another HU's activation is near 1.0 for the words "heed" and "hid" ("i" sounds). Another HU's activation has high values for the words "hod", "hoard", "hood" and "who'd" ("o"-words) and low but nonzero values

for "hard" and "heard". LOCOCODE supports feature detection.

Why no sparse code? The real-valued input components cannot be described precisely by the activations of the few feature detectors generated by LOCOCODE. Additional real-valued HUs are necessary for representing the missing information.

Better results with additional information. Hastie et al. also obtained additional, even slightly better results with an FDA/MARS variant: down to 39 % average error rate. It should be mentioned, however, that their data was subject to goal-directed preprocessing with splines, such that there were many clearly defined classes. Furthermore, to determine the input dimension, Hastie et al. used a special kind of generalized cross-validation error, where one constant was obtained by unspecified "simulation studies". Hastie and Tibshirani (1996) also obtained an average error rate of 38 % with discriminant adaptive nearest neighbor classification. About the same error rate was obtained by Flake (1998) with RBF networks and hybrid architectures. Also, recent experiments (mostly conducted during the time this paper has been under review) showed that even better results can be obtained by using additional context information to improve classification performance, e.g., Turney (1993), Herrmann (1997), and Tenenbaum and Freeman (1997). For an overview see Schraudolph (1998). It will be interesting to combine these methods with LOCOCODE.

Conclusion. Although we made no attempt at preventing classifier overfitting, we achieved excellent results. From this we conclude that the lococodes fed into the classifier already conveyed the "essential", almost noise-free information necessary for excellent classification. We are led to believe that LOCOCODE is a promising method for data preprocessing.

5 CONCLUSION

LOCOCODE, our novel approach to unsupervised learning and sensory coding, does not define code optimality solely by properties of the code itself but takes into account the information-theoretic complexity of the mappings used for coding and decoding. The resulting lococodes typically compromise between conflicting goals. They tend to be sparse and exhibit *low but not minimal* redundancy — if the costs of generating minimal redundancy are too high. Lococodes tend towards binary, informative feature detectors, but occasionally there are trinary or continuous-valued code components (where complexity considerations suggest such alternatives).

A general principle? According to our analysis LOCOCODE essentially attempts at describing single inputs with as few and as simple features as possible. Depending on the statistical properties of the input, this can result in either local, factorial, or sparse codes, although biologically plausible sparseness is the most common case. Unlike the objective functions of previous methods (e.g., Olshausen and Field 1996), however, LOCOCODE's does *not* contain an explicit term enforcing, say, sparse codes — sparseness or factoriality are not viewed as a good things *a priori*. This seems to suggest that LOCOCODE's objective may embody a general principle of unsupervised learning going beyond previous, more specialized ones.

Regularizers and unsupervised learning. Another way of looking at our results is this: there is at least one representative (FMS) of a broad class of algorithms (regularizer algorithms that reduce net complexity) which can do optimal feature extraction as a by-product. This reveils an interesting, previously ignored connection between two important fields: regularizer research, and ICA-related research.

Advantages. LOCOCODE is appropriate if single inputs (with many input components) can be described by few features computable by simple functions. Hence, assuming that visual data can be reduced to few simple causes, LOCOCODE is appropriate for visual coding. Unlike simple ICA, LOCOCODE (a) is not inherently limited to the linear case, and (b) does not need *a priori* information about the number of independent data sources. Even when the coding problem is linear and the number of sources is known, however, LOCOCODE can outperform other coding methods. This has been demonstrated by our LOCOCODE implementation based on FMS-trained autoassociators (AAs), which easily solves linear coding tasks that have been described as hard by other authors, and whose input causes are not perfectly separable by standard AAs, PCA, and

ICA. Furthermore, when applied to realistic visual data, LOCOCODE produces familiar on-center-off-surround receptive fields and biologically plausible sparse codes (standard AAs do not). Our experiments also demonstrate the utility of LOCOCODE-based data preprocessing for subsequent classification.

Limitations. FMS' order of computational complexity depends on the number of output units. For typical classification tasks (requiring few output units) it equals standard backprop's. In the AA case, however, the output's dimensionality grows with the input's. That's why large scale FMS-trained AAs seem to require parallel implementation. Furthermore, although LOCOCODE works well for visual inputs, it may be less useful for discovering input causes that can only be represented by high-compexity input transformations, or for discovering many features (causes) collectively determining single input components (as, e.g., in acoustic signal separation, where ICA does not suffer from the fact that each source influences each input component and none is computable by a low-complexity function).

Future work. Encouraged by the familiar lococodes obtained in our experiments with visual data we intend to move on to higher-dimensional inputs and larger receptive fields. This may lead to even more pronounced feature detectors like those observed by Schmidhuber et al. (1996). It will also be interesting to test whether successive LOCOCODE stages, each feeding its code into the next, will lead to complex feature detectors such as those discovered in deeper regions of the mammalian visual cortex. Finally, encouraged by our successful application to vowel classification, we intend to look at more complex pattern recognition tasks.

We also intend to look at alternative LOCOCODE implementations besides FMS-based AAs. Finally we would like to improve our understanding of the relationship between low-complexity codes, low-complexity art (see Schmidhuber, 1997b) and informal notions such as "beauty" and "good art".

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