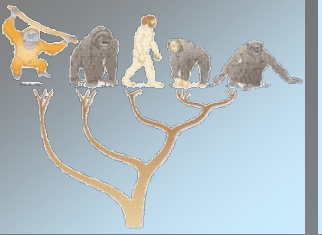
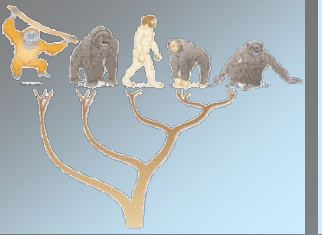


Sequence Analysis and Phylogenetics

Part 3

Sepp Hochreiter



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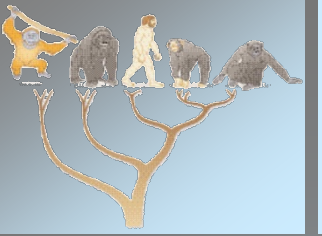
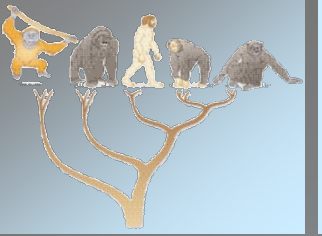


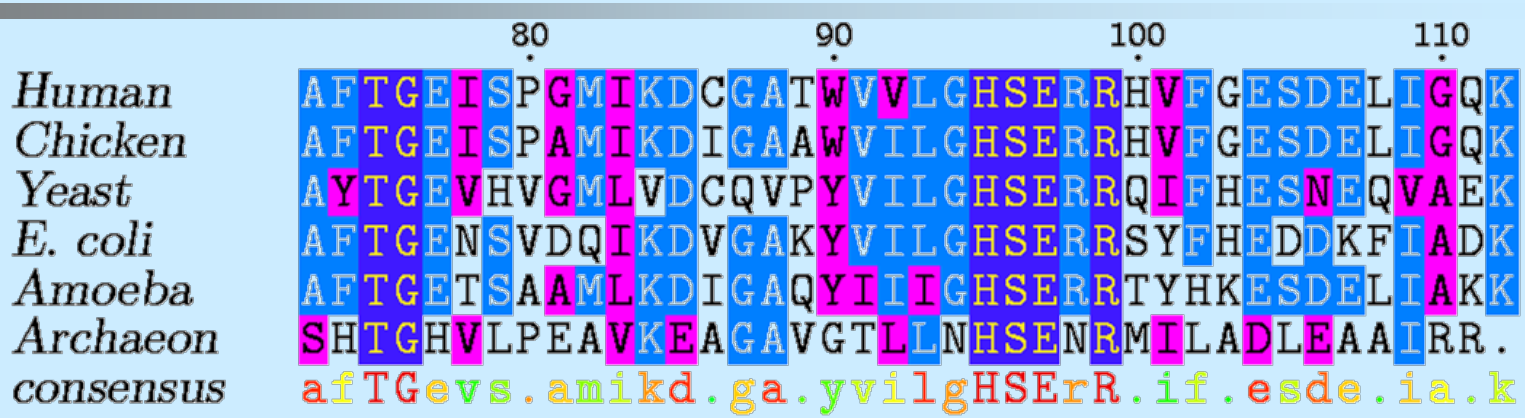
Motivation

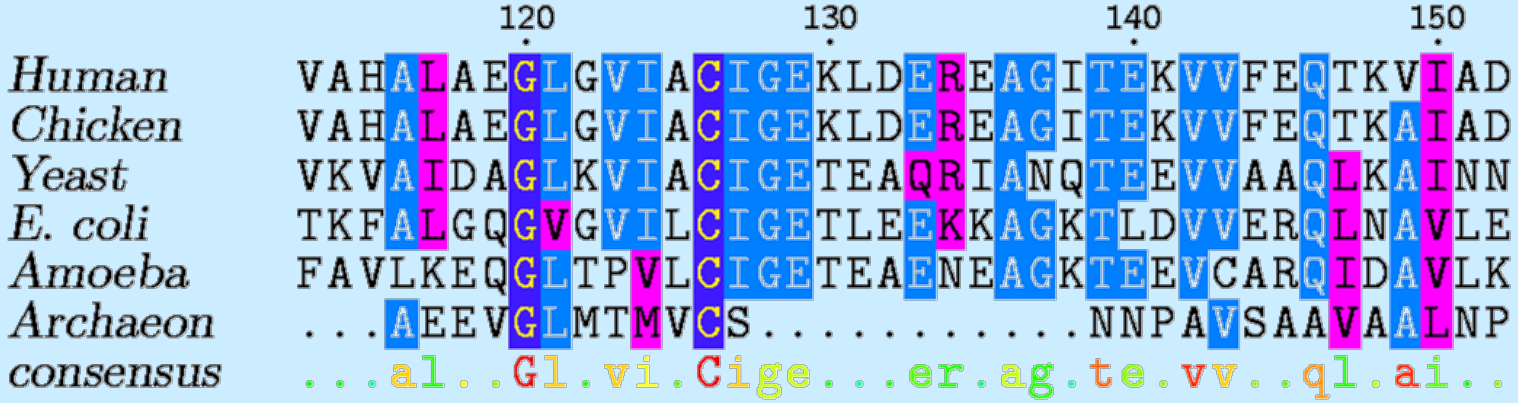
Compare more than two sequences: arranged sequences so that the amino acids for every the columns match as good as possible

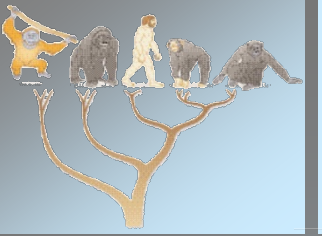
```
                    10        20        30
Human       APSRKFFVGGNWKMNGRKQSLGELIGTLNA...AKVPADT
Chicken     ...RKFFVGGNWKMNGDKKSLGELIHTLNG...AKLSADT
Yeast       .GAGKFVVGGNWKCNGTLASIETLTKGVAASVDAELAKKV
E. coli     ..ARTFFVGGNFKLNGSKQSIKEIVERLNT...ASIPENV
Amoeba      ..MRHPLVMGNWKLNGSRHMVHELVSNLRK..ELAGVAGC
Archaeon    AKLKEPIIAINFKTYIEATGKRALEIAKAA...EKVYKET
consensus   ...r.f.vggNwKlng.k.si.elv..l.a...a.v....

                    40        50        60        70
Human       EVVCAPPTAYIDFARQKLD.....PKIAVAAQNCYKVTNG
Chicken     EVVCGAPSIYLDFARQKLD.....AKIGVAAQNCYKVPKG
Yeast       EVIVGVPFIYIPKVQQILAGEANGANILVSAENAWTKS.G
E. coli     EVVICPPATYLDYSVSLVKK....PQVTVGAQNAYLKASG
Amoeba      AVAIAPPEMYIDMAKREAEG....SHIMLGAQNVNLNLSG
Archaeon    GVTIVVAPQLVDLRMIAES.....VEIPVFAQHIDPIKPG
consensus   eVvia.p..yid.....l.........i.vgAqn.y....G
```



Motivation

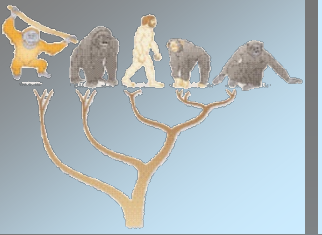






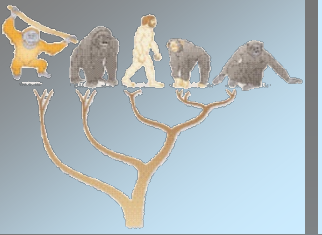
Motivation


```
                    160       170       180       190
Human       NV..KDWSKVVLAYEPVWAIGTGKTATPQQAQEVHEKLRG
Chicken     NV..KDWSKVVLAYEPVWAIGTGKTATPQQAQEVHEKLRG
Yeast       AISKEAWKNIILAYEPVWAIGTGKTATPDQAQEVHQYIRK
E. coli     EV..KDFTNVVVAYEPV.AIGTGLAATPEDAQDIHASIRK
Amoeba      TQGAAAFEGAVIAYEPVWAIGTGKSATPAQAQAVHKFIRD
Archaeon    DY.........VAVEPPELIGTGIPVSKAKPEVITN....
consensus   .v....w..vvlAyEPvwaIGTGktatp.qaqevh..ir.

                    200       210       220       230
Human       WLKSNVSDAVAQSTRIIYGGSVTGATCKELASQPDVDGFL
Chicken     WLKTHVSDAVAQSTRIIYGGSVTGGNCKELASQHDVDGFL
Yeast       WMTENISKEVAEATRIQYGGSVNPANCNELAKKADIDGFL
E. coli     FLASKLGDKAASELRILYGGSANGSNAVTFKDKADVDGFL
Amoeba      HIAK.VDANIAEQVIIQYGGSVNASNAAELFAQPDIDGAL
Archaeon    ..TVELVKKVNPEVKVLCGAGISTGEDVKKAIELGTVGVL
consensus   wl...v...va...rilyGgsv.ggn..ela...dvdGfL
```



Motivation

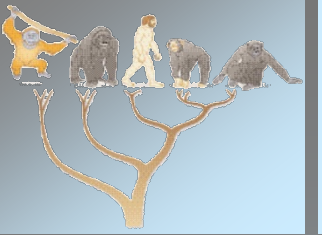

```
                    240
                    .
Human       VGGASLKP.EFVDIINAKQ.....
Chicken     VGGASLKP.EFVDIINAKH.....
Yeast       VGGASLDAAKFKTIINSVSEKL..
E. coli     VGGASLKP.EFVDIINSRN.....
Amoeba      VGGASLKADAFAVIVKAAEAAKQA
Archaeon    LASGVTKAKDPEKAIWDLVSGI..
consensus   vggaslk..ef..iin........
```



Motivation

Multiple sequence alignment is used to

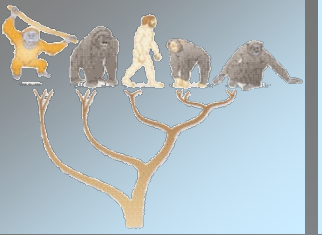
- ↪ detect remote homologous regions
- ↪ detect motifs (regular patterns) in protein families
- ↪ detect conserved regions or positions (disulfide bonds)
- ↪ detect structural blocks like helices or sheets
- ↪ construct phylogenetic trees
- ↪ construct a profiles (search or averages)
- ↪ sequence genomes by superimposing fragments (nucleotides)
- ↪ cluster proteins according to similar regions



Scoring and Similarity

Similarity measures can be based on:

- the similarity of all sequences to a reference sequence
- the similarities between evolutionary adjacent sequences
- all pairwise similarities



Consensus and Entropy

consensus sequence: obtained if for each column in the alignment

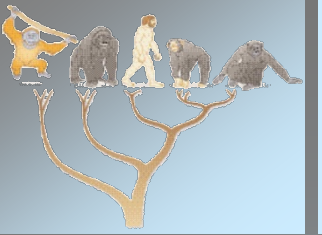
1. the most frequent amino acid or
2. the amino acid which has the highest score to all other amino acids

is chosen

consensus score: sum of the pairwise score between sequences and the consensus sequence

generalized by profiles instead of sequences

profile: relative frequency instead of most frequent



Consensus and Entropy

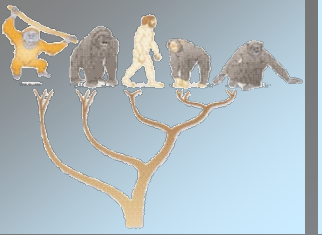
high entropy of the letter distribution: all letter are equally probable
zero entropy: one letter in the column

good alignment correlates with a low accumulative entropy

entropy score:

$$-\sum_i \sum_a f_{i,a} \log f_{i,a}$$

$f_{i,a}$: relative frequency of letter a in column i

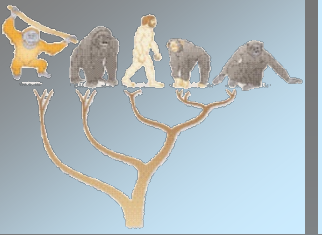


Tree and Star Score

To count the number of mutations only those pairs should be compared which are evolutionary adjacent

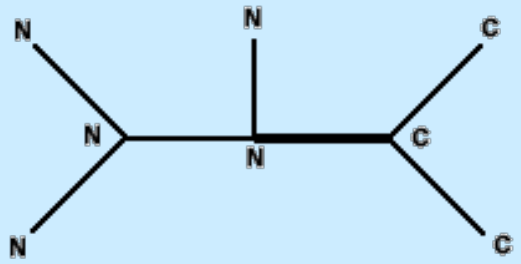
E
E
E
E
D
D
D
D

evolutionary adjacent sequences are represented through a phylogenetic tree, which must be constructed

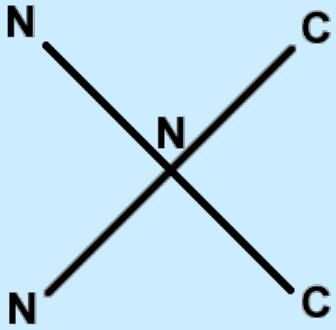


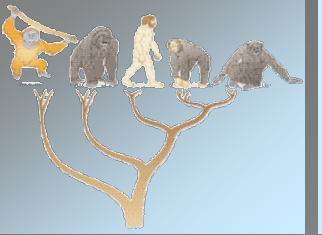
Tree and Star Score

NNN
NNN
NNN
NNC
NCC



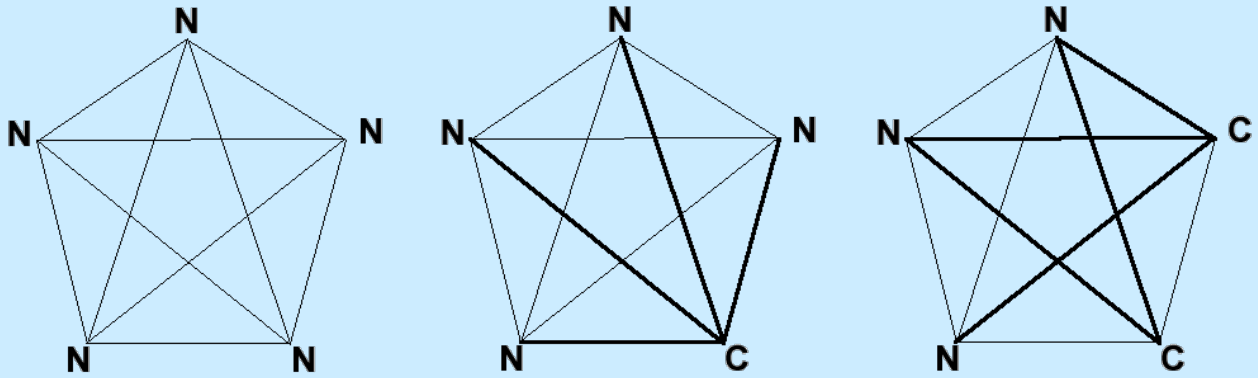
phylogenetic star: one sequence is considered as ancestor





Weighted Sum of Pairs

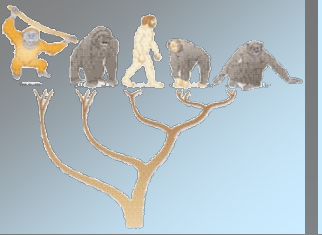
weighted sum of pairs: all pairwise comparisons



alignment length: L
number sequences: N

$$\sum_{i=1}^{L} \sum_{l=1}^{N-1} \sum_{j=l+1}^{N} w_{l,j} \; s\left(x_{i,l}, x_{i,j}\right)$$

weights: reduce the influence of closely related sequences



Weighted Sum of Pairs

Disadvantage: score relatively decreases with increasing N for conservative regions; but larger N means more conservative

C	C
C	C
C	C
C	C
C	D

N Cs vs. (N-1) Cs and one D

$$S_{\text{old}} = \frac{N\ (N-1)}{2} s(C,C)$$

$$S_{\text{new}} = \frac{N\ (N-1)}{2} s(C,C) - (N-1)s(C,C) + (N-1)s(C,D)$$

$$\frac{S_{\text{old}} - S_{\text{new}}}{S_{\text{old}}} = \frac{2\ (N-1)\ s(C,C) - 2\ (N-1)\ s(C,D)}{N\ (N-1)\ s(C,C)} =$$

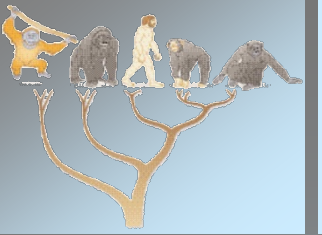
$$\frac{2}{N}\left(1 - \frac{s(C,D)}{s(C,C)}\right)$$

the larger N, the smaller the difference (paradox!)

reasonable scoring matrices:

$$s(C,D) < s(C,C)$$

$$\left(1 - \frac{s(C,D)}{s(C,C)}\right) > 0$$

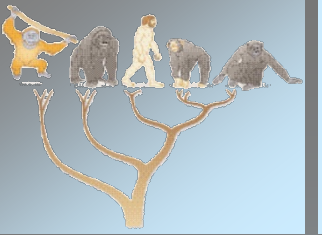


Weighted Sum of Pairs

contra-intuitive: a new letter in a column of 100 equal letters is more surprising as a new letter in a column of 3 equal letters

Information gain: $-\log f_{i,a} = \log(N)$

Gaps: as for pairwise algorithms, linear gaps more efficient



Multiple Alignment Algorithms

multiple alignment optimization problem: NP-hard

Exact solution: only 10 to 15 sequences

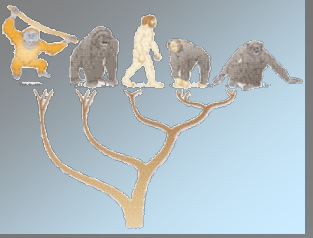
algorithm classes:

➥ global and progressive methods: MSA, COSA, GSA, clustalW, TCoffee

➥ iterative and search algorithms: DIALIGN, MultAlin, SAGA, PRRP, Realigner

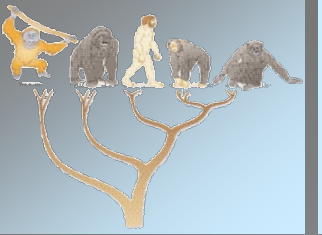
➥ local methods (motif/profile): eMotif, Blocks, Dialign, Prosite, HMM, Gibbs sampling

➥ divide-and-conquer algorithms: DCA, OMA



Multiple Alignment Algorithms

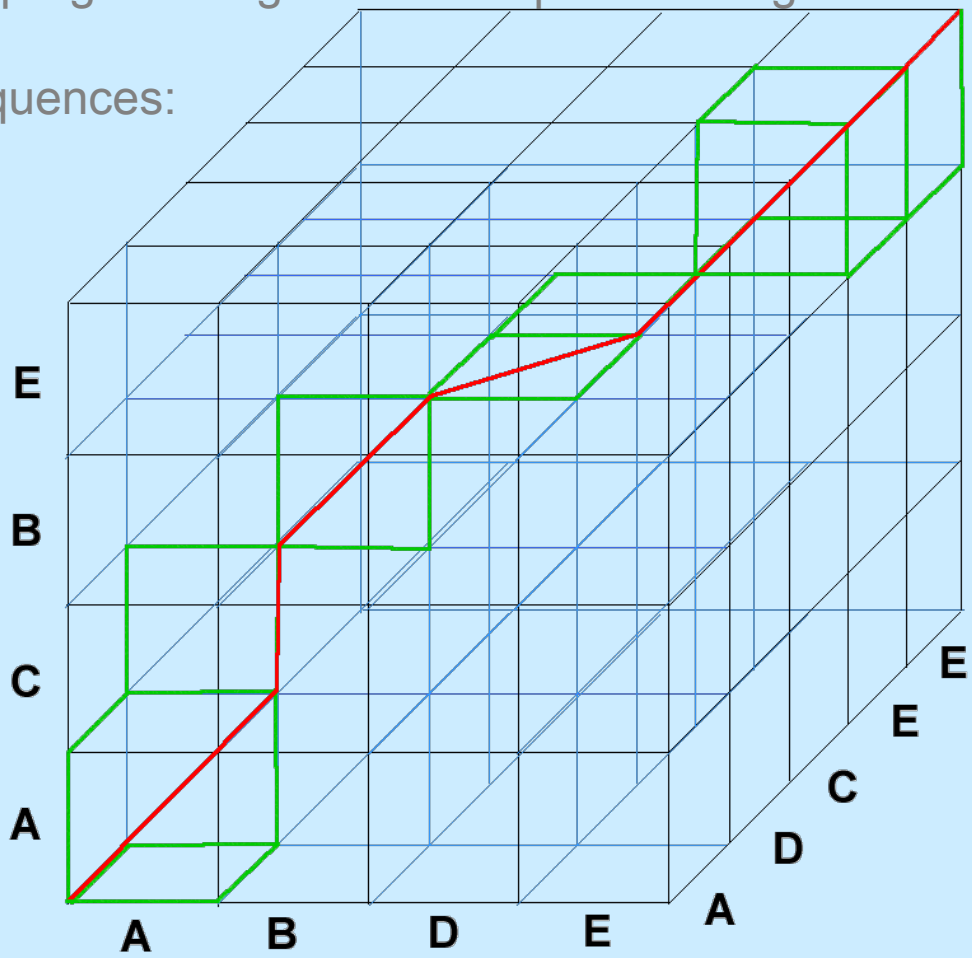
Global progressive alignments methods		
CLUSTALW	ftp://ftp.ebi.ac.uk/pub/software	Thompson et al. (1994/97) Higgins et al. (1996)
MSA	http://www.psc.edu/ http://www.ibc.wustl.edu/ibc/msa.html ftp://fastlink.nih.gov/pub/msa	Lipman et al. (1989) Gupta et al. (1995)
PRALINE	http://mathbio.nimr.mrc.ac.uk/ ~jhering/praline	Heringa (1999)
Iterative and search algorithms		
DIALIGN segment alignment	http://www.gsf.de/biodv/dialign.html	Morgenstern et al. (1996)
MultAlin	http://protein.toulouse.inra.fr/multalin.html	Corpet (1988)
PRRP progressive global alignment	ftp://ftp.genome.ad.jp/ pub/genome/saitamacc	Gotoh (1996)
SAGA genetic algorithm	http://igs-server.cnrs-mrs.fr/~cnotred/ Projects_home_page/saga_home_page.html	Notredame and Higgins (1996)
Local alignments / motif / profile		
Aligned Segment Statistical Eval. Tool (Asset)	ftp://ncbi.nlm.nih.gov/pub/neuwald/asset	Neuwald and Green (1994)
BLOCKS	http://blocks.fhcrc.org/blocks/	Henikoff and Henikoff (1991, 1992)
eMOTIF	http://dna.Stanford.EDU/emotif/	Nevill-Manning et al. (1998)
GIBBS (Gibbs sampler)	ftp://ncbi.nlm.nih.gov/ pub/neuwald/gibbs9_95/	Lawrence et al. (1993) Liu et al. (1995) Neuwald et al. (1995)
HMMER hidden Markov model	http://hmmer.wustl.edu/	Eddy (1998)
MACAW	ftp://ncbi.nlm.nih.gov/pub/macaw	Schuler et al. (1991)
MEME (EM method)	http://meme.sdsc.edu/meme/website/	Bailey and Elkan (1995) Grundy et al. (1996, 1997) Bailey and Gribskov (1998)
Profile (UCSD)	http://www.sdsc.edu/projects/profile/	Gribskov and Veretnik (1996)
SAM hidden Markov model	http://www.cse.ucsc.edu/ research/comp/bio/sam.html	Krogh et al. (1994) Hughey and Krogh (1996)



Exact Methods

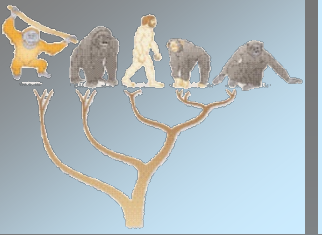
MSA (Lippman et al., 1989, Gupa et al., 1995): generalizes the dynamic programming ideas from pairwise alignment

three sequences:




```
- - E
EEE
- - C
D - D
BB -
- C -
AAA
```

```
A-BD-E-
ACB--E-
A--DCEE
```



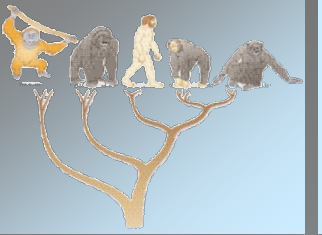
Exact Methods

memory and computational complexity: exponentially with N

Gupa et al., 1995: pairwise alignments constrain the path and not the whole hypercube must be filled

MSA (Gupa):

1. compute all pairwise alignment scores $S_{k,l}$
2. predict a phylogenetic tree based on the pairwise scores
3. compute pairwise weights based on the tree
4. construct a temporary multiple alignment with score S_t
5. Compute $B_{k,l}$, a lower bound on $S[k,l]$, the score of the projection of the optimal multiple alignment to k and l
6. Compute space constraints similar to the Baum-Welch
7. compute the optimal alignment on the constraint cube; Dijkstra's shortest path algorithm for nonnegative edges; priority queue; non-negativity guarantees monotone increasing **costs**
8. compare the weight in the alignment with the maximal weight



Exact Methods

last step compares actual and maximal weight, if actual is larger then a better alignment may be possible, larger maximal weight means more computational costs

Carillo-Lipman bound:

$$B_{k,l} = S_t + S_{k,l} - \sum_{i,j} S_{i,j}$$

$$S \geq S_t$$

$$\Leftrightarrow \sum_{i,j} S[i,j] \geq S_t$$

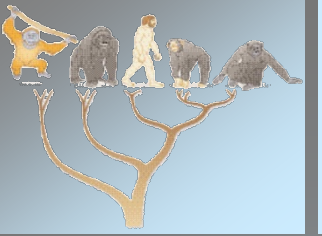
$$\Rightarrow \sum_{(i,j)\neq(k,l)} S_{i,j} + S[k,l] \geq S_t$$

$$\Leftrightarrow S[k,l] \geq S_t - \sum_{(i,j)\neq(k,l)} S_{i,j}$$

$$\Leftrightarrow S[k,l] \geq S_t + S_{k,l} - \sum_{i,j} S_{i,j}$$

$$\Leftrightarrow S[k,l] \geq B_{k,l}$$

$$S[k,l] \leq S_{k,l}$$
$$S_t \leq S$$



Exact Methods

MSA improved by the $\mathcal{A}^*$ algorithm (Lermen and Reinert, 1997)

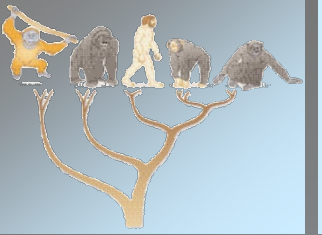
A^*-algorithm

Lower bound

Input: `graph` (the graph), `start` (start node), `goal` (goal node), `h(s)` approximation of the distance of node `s` to the goal, `S` (priority queue), `N` (list of visited nodes)
Output: list `P` of the shortest path

```
BEGIN FUNCTION
  insert (start,S)
  while not isEmpty(S) do
    current_node = pop(S)
    if current_node in N then {no path from start to goal}
      return "no path"
    end if
    insert (current_node, N)
    if current_node = goal then
      reconstruct_shortest_path(start,goal, graph)
    else {find all nodes accessible from current node}
      successors = expand(current_node, graph)
      save_predecessor_in_graph(current_node, graph)
      for all s in successors do {save node which lead to s}
        predecessor(s) = current_node {compute and store costs}
        cost(s) = cost(current_node) + edge(graph,current_node,s)
        all_cost(s) = cost(s) + h(s)
        insert(s,S) {according to all_cost(s)}
      end for
    end if
  end while
  return "no path found"
END FUNCTION
```

```
BEGIN SUBFUNCTION {shortest path P as list}
 reconstruct_shortest_path (start, node, graph)
  if node not= start then
    push(node, P) {get predecessor}
    predecessor = getPredecessor(node, graph)
    reconstruct_shortest_path (start, predecessor, graph)
  else
    return P
  end if
END SUBFUNCTION
```



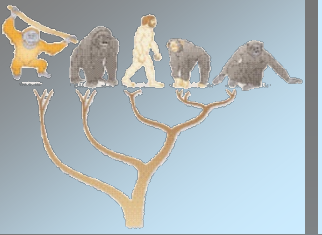
Exact Methods

MSA: weighted sum of pairs and a linear gap penalty
Weight: difference pairwise and projected multiple alignment (larger difference means higher weight)

similar sequences: pull the multiple alignment towards them which down-weights them

weights through the phylogenetic tree remove weights between distant sequences

Summing up all the weights: overall divergence of the sequences



Progressive Methods

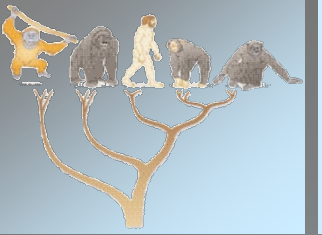
Progressive methods are the most popular methods for multiple alignment: ClustalW (Thomson,Higgins,Gibson, 1994) and TCoffee (Notredame, Higgins, Heringa, 2000)

ClustalW and TCoffee:

- perform pairwise alignment for each pair
- weight matrix: one minus the ratio of perfect matches
- construct a phylogenetic tree (Neighbor-Joining method)
- alignments between pairs sequences/alignments (start with closest distance); alignments are propagated through the tree

Initial alignments may be found through local alignment

phylogenetic tree supplies the weighting factors



Progressive Methods

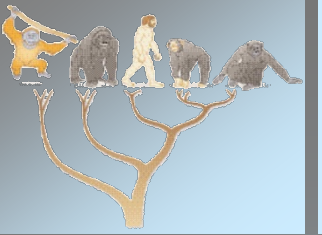
Disadavantage progressive methods:

- local minima
- same scoring matrix for close and remote related sequences and same gap parameters

ClustalW

gap penalties context dependent:

- gaps in hydrophobic regions are more penalized
- gaps which are within eight amino acids to other gaps are more penalized
- gaps in regions of other gaps have lower gap opening penalty
- gap penalties are amino acid dependent



Progressive Methods

scoring matrices are adapted:

➥ scoring matrix from the PAM or the BLOSUM families

sequences are weighted through a phylogenetic tree:

➥ similar sequences lower weights (unbalanced data sets)

➥ phylogentic tree weights with w_i as the weight of sequence i

$$\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} w_i \; w_j \; s(i,j)$$

adaptive phylogenetic tree:

➥ insufficient scores change the tree

initial gap penalty parameters:

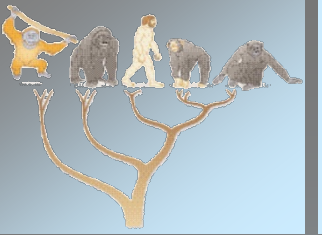
➥ according to scoring matrix

➥ similarity of the sequences (% identity)

➥ length of the sequences (log of the shorter sequences is added)

➥ difference of the length to avoid gaps in the shorter sequence

$$\cdot \; (1 \; + \; |\log(n/m)|)$$

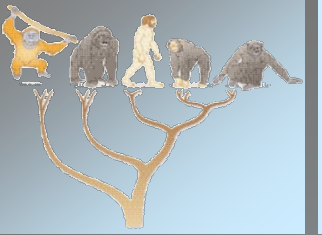


Progressive Methods

TCoffee (Tree based Consistency Objective Function For alignmEnt Evaluation) often better alignment than clustalW

TCoffee work as follows:

- ➥ libraries of pairwise aligments based on both global (clustalW) and local (FASTA) alignments (combination is more reliable)
- ➥ library weights are computed according to % identity
- ➥ libraries are combined and extended; arithmetic mean of weights; extension by aligning two sequences through a third sequence
- ➥ progressive alignment with a distance based on extended library



Other Methods

Center Star Alignment

center sequence $\bar{i}$: $\bar{i} = \arg\min_i \sum_j C_{i,j}$

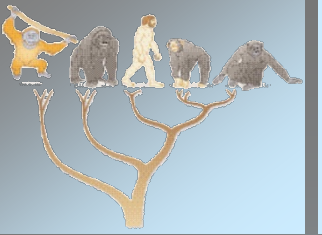
pairwise alignment costs $C_{i,j}$

$\bar{i} = 1$

new sequence is added to the set of aligned sequences by
a pairwise alignment to the center sequence introducing new gaps

Therefore for center star cost C with projection $C(i,j)$:

$C(1,j) = C_{1,j}$



Other Methods

Gusfield, 1993: cost is less then twice the optimal cost, if

$$C(i,i) = 0 \quad \text{and} \quad C(i,j) \leq C(i,k) + C(k,j)$$

scoring matrix s with

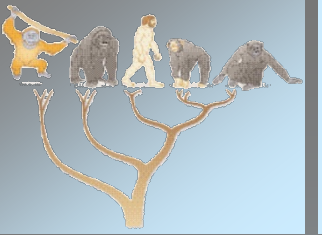
$$s(-,-) = 0$$
$$s(-,i) < 0$$
$$s(k,k) \geq s(i,k) + s(k,j) - s(i,j)$$

A	B		A	B
|	|	>	|	|
A	C		C	A

Then $C(i,j) = S_{i,i} - 2\,S_{i,j} + S_{j,j}$ fulfills above conditions

Then the condition $C(i,j) \leq C(i,k) + C(k,j)$ is equivalent to

$$S_{i,i} - 2\,S_{i,j} + S_{j,j} \leq S_{i,i} - 2\,S_{i,k} + S_{k,k} + S_{k,k} - 2\,S_{k,j} + S_{j,j}$$
$$\Leftrightarrow \quad S_{i,j} \geq S_{i,k} + S_{k,j} - S_{k,k}$$
$$\Leftrightarrow \quad S_{k,k} \geq S_{i,k} + S_{k,j} - S_{i,j}$$



Other Methods

align i to k and j to k then align i, j, and k based on the pairwise alignments, the alignment has a gap if a gap was in one alignment

S is score of the multiple alignment

Per construction: $S[i,k] = S_{i,k}$, $S[k,j] = S_{k,j}$ and $S[k,k] = S_{k,k}$

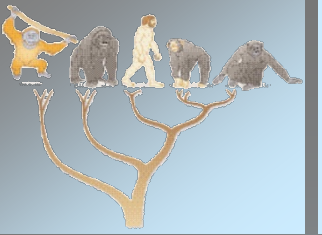
Componentwise holds: $s(i,j) \geq s(i,k) + s(k,j) - s(k,k)$

Therefore $S[i,j] \geq S[i,k] + S[k,j] - S[k,k]$ and

$$S[i,j] \geq S_{i,k} + S_{k,j} - S_{k,k}$$

inequality to show follows from $S_{i,j} \geq S[i,j]$

→triangle inequality for costs shown by previous equivalences



Other Methods

idea of the proof of Gusfield center sequence alignment with cost C and the optimal cost C^*

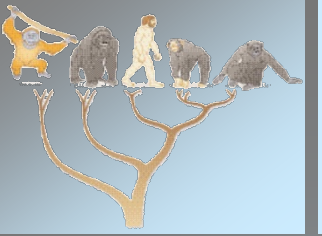
$$C = \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} C(i,j) \leq$$

$$\sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} C(i,1) + C(1,j) = 2\,(N-1) \sum_{i=2}^{N} C_{i,1}$$

$$C^* = \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} C^*(i,j) \geq \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} C_{i,j} \geq$$

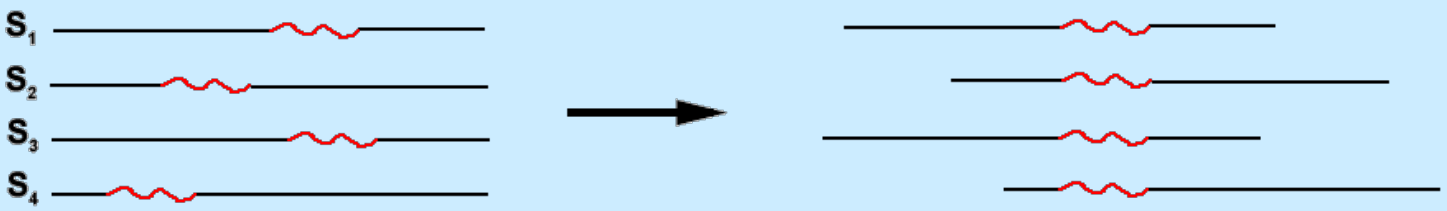
$$\sum_{i=1}^{N} \sum_{j=2}^{N} C_{i,1} = N \sum_{i=2}^{N} C_{i,1}$$

$$\Rightarrow \quad \frac{C}{C^*} \leq \frac{2(N-1)}{N} \leq 2$$

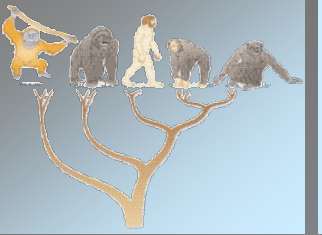


Other Methods

Motifs or pattern can be superimposed for alignment landmarks



Profiles and blocks can be derived from multiple alignments



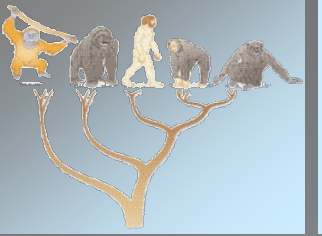
Other Methods

SAGA (Sequence Alignment by Genetic Algorithm): genetic algorithm

MSASA (Multiple Sequence Alignment by Simulated Annealing): simulated annealing

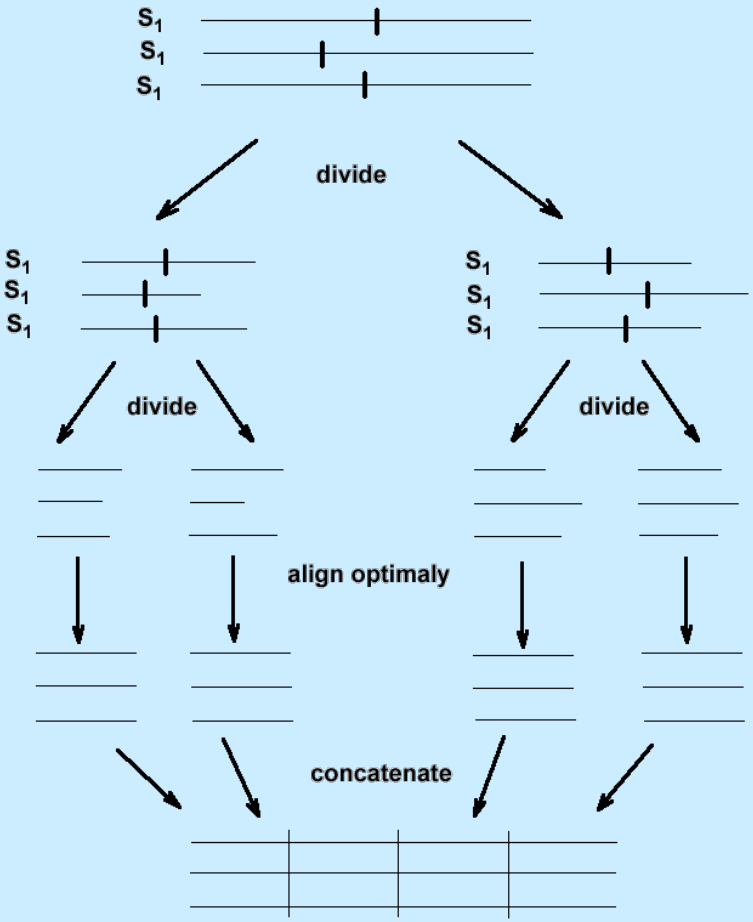
Gibbs sampling

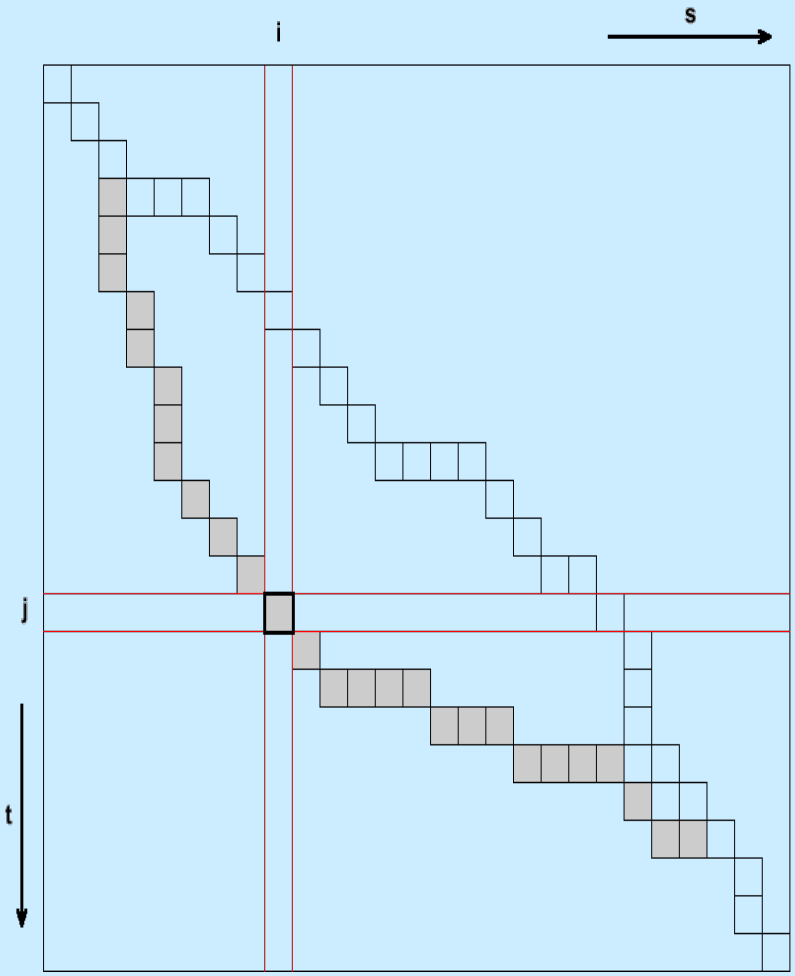
HMMs (hidden Markov models) can be used to find motifs

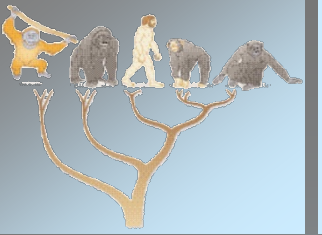


Other Methods

Divide-and-conquer Algorithms







Profiles and PSSMs

Profiles and Position Specific Scoring Matrices

Assumptions:

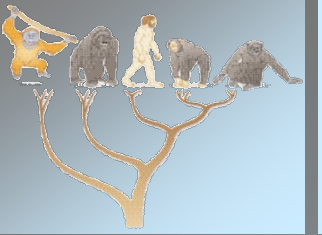
- $\boldsymbol{x}$ is i.i.d. in its elements according to p_x
- n the length of $\boldsymbol{x}$ is large
- expected letter score for random sequences $\sum_i \; p_x(i) \; s(i) \; < \; 0$
- exist i for which $\; s(i) \; > \; 0$

$$S_n \; = \; \sum_{i=1}^{n} \; s(i)$$

centered value: $\tilde{S}_n \; = \; S_n \; - \; \frac{\ln n}{\lambda}$

$$P\left(\tilde{S}_n > S\right) \; \approx \; 1 - \exp\left(-K \; e^{-\lambda \; S}\right) \; \approx \; K \; e^{-\lambda \; S}$$

$$\sum_i p_x(i) \; \exp(\lambda \; s(i)) \; = \; 1$$



Profiles and PSSMs

q_i: frequency of a letter a_i in a column of a multiple alignment

for sufficient high scoring segments

$$\lim_{n \to \infty} q_i = p_x(i) \exp(\lambda s(i))$$

$$\Rightarrow \quad s(i) = \ln\left(\frac{q_i}{p_x(i)}\right) / \lambda$$

“Position Specific Scoring Matrices” (PSSMs) or profiles

new sequence: high scores mean similar alignment sequences